GRADE 8
RESOURCE GUIDE
PART 2

MATHLINKS: GRADE 8
RESOURCE GUIDE: PART 2

Table of Contents	**Pages**

THE STANDARDS FOR MATHEMATICAL PRACTICE

In addition to the mathematical topics you will learn about in this course, your teacher will help you become better at what are called the Mathematical Practices. The Standards for Mathematical Practice describe a variety of processes and strategies to help you to be more mathematically proficient and fluent students.

One way to think about the practices is in groupings.

HABITS OF MIND

MP1 Make sense of problems and persevere in solving them
MP6 Attend to precision

REASONING AND EXPLAINING

MP2 Reason abstractly and quantitatively
MP3 Construct viable arguments and critique the reasoning of others

MODELING AND USING TOOLS

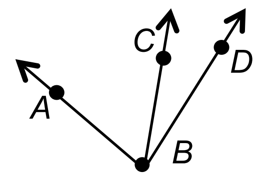
MP4 Model with mathematics
MP5 Use appropriate tools strategically

SEEING STRUCTURE AND GENERALIZING

MP7 Look for and make use of structure
MP8 Look for and make use of repeated reasoning

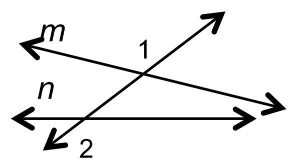
WORD BANK

<table>
<tr><th>word or phrase</th><th>definition</th></tr>
<tr><td>acute angle</td><td>An <u>acute angle</u> is an angle whose measure is less than 90°. See <u>angle</u>.</td></tr>
<tr><td>adjacent angles</td><td>Two angles are <u>adjacent</u> if they have the same vertex and share a common ray, and they lie on opposite sides of the common ray.



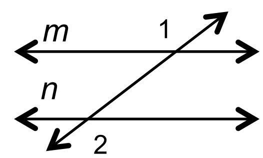
Example: $\angle ABC$ and $\angle CBD$ are adjacent angles.</td></tr>
<tr><td>alternate exterior angles</td><td>When two lines in a plane are cut by a transversal, two angles on opposite sides of the transversal and outside the two lines are referred to as <u>alternate exterior angles</u>. When parallel lines are cut by a transversal, alternate exterior angles have the same measure.

Examples:



Line *m* is not parallel to line *n*.

$\angle 1$ and $\angle 2$ are alternate exterior angles.

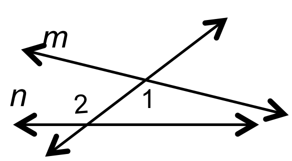


Line *m* is parallel to line *n.*

$\angle 1$ and $\angle 2$ are alternate exterior angles.

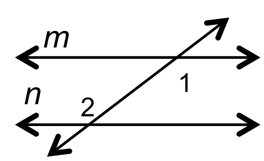
$|\angle 1| = |\angle 2|$</td></tr>
<tr><td>alternate interior angles</td><td>When two lines in a plane are cut by a transversal, two angles on opposite sides of the transversal and between the two lines are referred to as <u>alternate interior angles</u>. When parallel lines are cut by a transversal, alternate interior angles have equal measure.

Examples:



Line *m* is not parallel to line *n*.

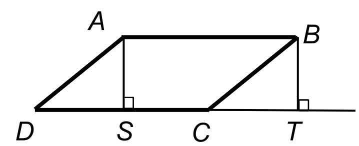
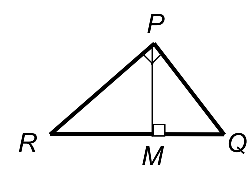
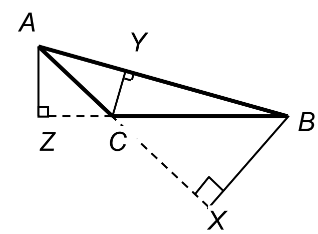
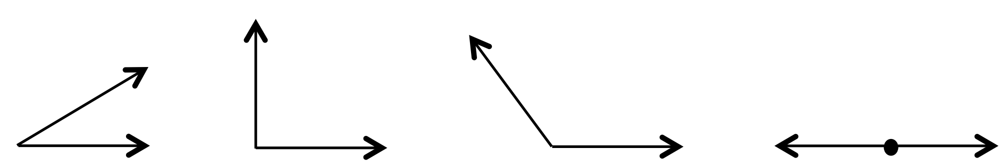
$\angle 1$ and $\angle 2$ are alternate interior angles.

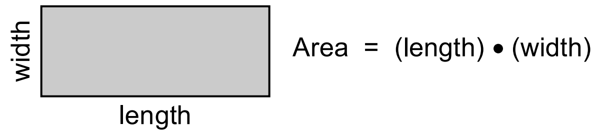
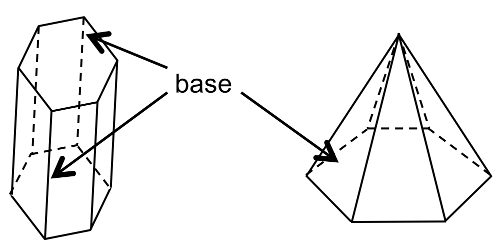


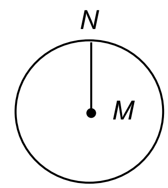
Line *m* is parallel to line *n.*

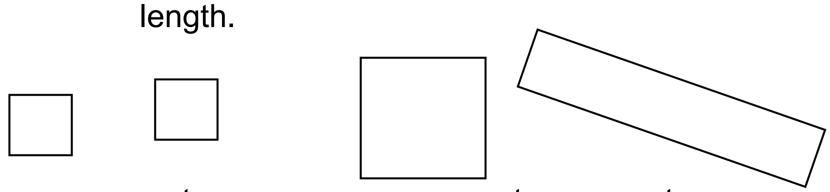
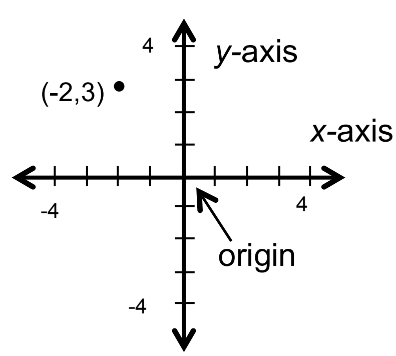
$\angle 1$ and $\angle 2$ are alternate interior angles.

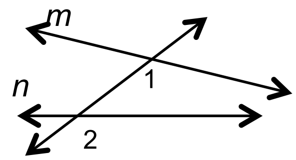
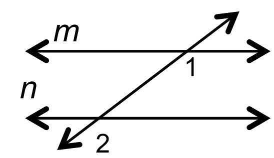
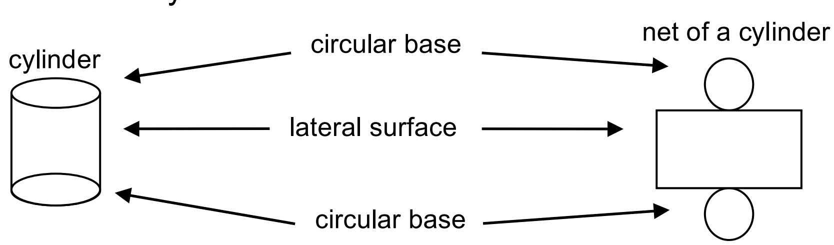
$|\angle 1| = |\angle 2|$</td></tr>
</table>

altitude of a parallelogram	An altitude of a parallelogram (on a given base) is a line segment perpendicular to the base and connecting a point on the base (extended if necessary) to a point on the opposite side. The height of the parallelogram is the length of the altitude. The altitude meets both the (extended) base and the opposite side at right angles. Example: In ▱$ABCD$, both $\overline{AS}$ and $\overline{BT}$ are altitudes with respect to the base $\overline{CD}$. 
altitude of a triangle	An altitude of a triangle is a line segment connecting one vertex to a point on the line through the other two vertices, meeting the line at a right angle. The height of the triangle is the length of the altitude. A triangle has three altitudes, each corresponding to a height. Example: In ΔPQR: $\overline{PM}$ is the altitude on $\overline{RQ}$ $\overline{RP}$ is the altitude on $\overline{PQ}$ $\overline{PQ}$ is the altitude on $\overline{RP}$  Example: In ΔABC: $\overline{CY}$ is the altitude on $\overline{AB}$ $\overline{AZ}$ is the altitude on $\overline{BC}$ $\overline{BX}$ is the altitude on $\overline{AC}$ 
angle	An angle is a geometric shape formed by two (distinct) rays that share a common endpoint (the vertex of the angle). To each angle is assigned a degree measure, which is a measure of the size of the angle and which is between 0 and 180 degrees. • An acute angle is an angle whose measure is less than 90°. • A right angle is an angle whose measure is exactly 90°. • An obtuse angle is an angle whose measure is between 90° and 180°. • A straight angle is an angle whose measure is 180°. The sides of a straight angle are opposite rays that form a straight line. 

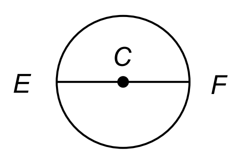
area	The <u>area</u> of a two-dimensional figure is a measure of the size of the figure, expressed in square units. The <u>area of a rectangle</u> is the product of its length and its width.  Example: If a rectangle has a length of 12 inches and a width of 5 inches, its area is (5)(12) = 60 square inches.
area of a circle	The <u>area of a circle</u> refers to the area of the figure (disk) enclosed by the circle. The area of a circle of radius r is πr^2 . See <u>area</u>, <u>circle</u>, <u>disk</u>.
association	In statistics, an <u>association</u> between two variables is a relationship between the variables, so that the variables are statistically dependent. In the case of numerical variables, if the relationship is linear, we refer to a <u>linear association</u> between the variables.
axis of symmetry	A line L is an <u>axis of symmetry</u> for a figure if the figure coincides with its own reflection through L.
base	The <u>base</u> of a figure is a predesignated side or face of the figure. The base is usually regarded as the “bottom” of the figure, on which it is standing. The “top” of a figure is sometimes also referred to as a base if it is congruent and parallel to the “bottom.” 
bivariate data	<u>Bivariate data</u> is data that has two variables. Bivariate data can be represented by ordered pairs. Example: A list of country of origin and batting average for each major league baseball player is a bivariate data set with one categorical variable and one numerical variable.

bivariate numerical data	Bivariate numerical data is data that has two numerical variables. Bivariate numerical data can be represented by a scatter plot, so that the relationship (if any) between the variables is more easily seen. Example: A list of heights and weights for each player on a football team is a bivariate numerical data set.
categorical data	Categorical data is data sorted into categories, such as colors, ranges of measurements, or other attributes of the data. Generally, there are only finitely many categories.
categorical variable	A categorical variable is a function that assigns to each member of the population a category. Example: The eye colors of students in a school determines a categorical variable, which assigns to each student the color of the student's eyes.
circle	A circle is a closed curve in a plane consisting of all points at a fixed distance (the radius) from a specified point (the center). Example: The center is at M and the radius is the length of the line segment from M to N. 
circumference	The circumference of a circle is the length of the circle, that is, the distance around it. The circumference of a circle of radius r is $C = 2\pi r$.
complementary angles	Two angles are complementary if the sum of their measures is 90°. Example: Two angles that measure 30° and 60° are complementary.
cone	A circular cone is a figure in space consisting of a circle in a plane (called the base of the cone), a point off the plane (called the vertex of the cone), and all the straight line segments joining the vertex to the base. If the line joining the vertex of the cone to the center of its base is perpendicular to the base, the cone is a right circular cone. Otherwise it is an oblique circular cone.  right circular cone oblique circular cone

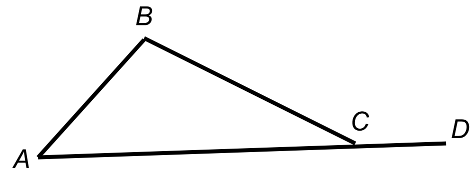
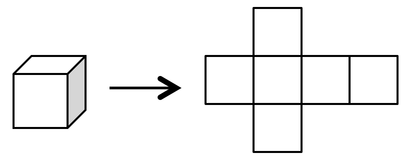
congruent figures	Two figures in the plane are <u>congruent figures</u> if the second can be obtained from the first by a sequence of translations, rotations, and reflections. Example: Two squares are congruent if they have the same side-length.  congruent not congruent
conjecture	A <u>conjecture</u> is a statement that is proposed to be true, but has not been proven to be true nor to be false. Example: After creating a table of sums of odd numbers such as 1 + 3 = 4, 1 + 5 = 6, 5 + 7 = 12, 3 + 9 = 12, etc., we may make a conjecture that the sum of any two odd numbers is an even number. This conjecture can be proven to be true.
converse of the Pythagorean Theorem	The <u>converse of the Pythagorean Theorem</u> states that if the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle. See <u>Pythagorean Theorem</u>. Example: If the lengths of the sides of a triangle are 3, 4, and 5 units respectively, then the triangle is a right triangle, because $3^2 + 4^2 = 5^2$.
coordinate plane	A <u>coordinate plane</u> is a (Euclidean) plane with two perpendicular number lines (<u>coordinate axes</u>) meeting at a point (the <u>origin</u>). Each point *P* of the coordinate plane corresponds to an ordered pair (*a*,*b*) of numbers, called the <u>coordinates</u> of *P*, determined by where the lines through *P* parallel to each coordinate axis meet the other coordinate axis. Example: The coordinate axes are often referred to as the <u>*x*-axis</u> and the <u>*y*-axis</u> respectively. Points on the *x*-axis have coordinates (*a*, 0), and points on the *y*-axis have coordinates (0, *b*). The origin has coordinates (0, 0). 

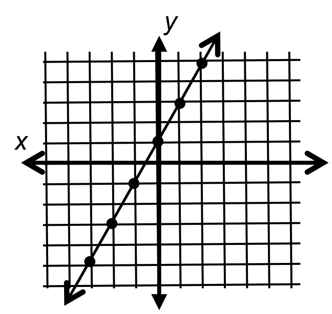
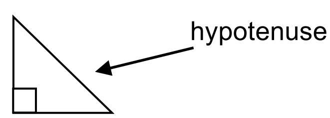
corresponding angles	When two lines in a plane are cut by a transversal, two angles that appear on the same side of the transversal in the same relative location are referred to as <u>corresponding angles</u>. When parallel lines are cut by a transversal, corresponding angles have the same measure. Examples:  Line *m* is not parallel to line *n*. $\angle 1$ and $\angle 2$ are corresponding angles.  Line *m* is parallel to line *n*. $\angle 1$ and $\angle 2$ are corresponding angles. $\lvert\angle 1\rvert = \lvert\angle 2\rvert$
cube of a number	The <u>cube of a number</u> n is the number $n^3 = n \bullet n \bullet n$. Example: The cube of -5 is -125, since $(-5)^3 = (-5)(-5)(-5) = -125$.
cube root	The <u>cube root</u> of a number n is the number whose cube is equal to n. That is, the cube root of n is the value of x such that $x^3 = n$. The cube root of n is written $\sqrt[3]{n}$. Example: The cube root of -125 is -5, because $(-5)^3 = -125$.
cylinder	A (right circular) <u>cylinder</u> is a figure in three-dimensional space that has two parallel circular bases. These circles are connected by a curved surface, called the <u>lateral surface</u>, which is a "rolled up" rectangle. Example: Most soup cans have the shape of a right circular cylinder. 

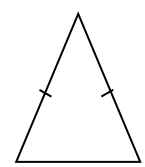
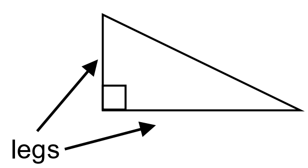
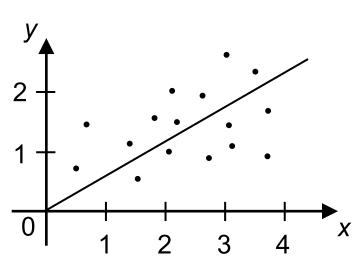
data displays	Data displays are ways of visually presenting data to make it more understandable. Example: Data displays include charts, tables, and a variety of graphical representations such as pictographs, circle graphs, bar graphs, line graphs, histograms, stem-and-leaf plots, box plots, and scatter plots.
data set	A data set is a collection of pieces of information, often numbers, obtained from observation, questioning, or measuring. Example: The following data set was collected in a survey of 10 students about the number of siblings they have: {2, 0, 2, 1, 4, 3, 1, 1, 2, 1}
decimal	A decimal is an expression of the form *n.abc* … , where *n* is a whole number written in standard form, and *a, b, c,* … are digits. Each decimal represents a unique nonnegative real number and is referred to as a decimal expansion of the number. Example: The decimal expansion of $\frac{4}{3}$ is 1.333333… . Example: The decimal expansion of π is 3.14159… .
decimal expansion	The decimal expansion (or decimal representation) of a nonnegative real number a is a decimal that represents a. The decimal expansion of $-a$ is minus the decimal expansion of a. Rational numbers with terminating decimal expansions have a second decimal expansion with repeating 9's. All other real numbers have a unique decimal expansion. Example: The decimal expansion of $\frac{4}{3}$ is 1.333333…. = $1.\overline{3}$. Example: The decimal expansion of $-\frac{4}{3}$ is -1.333333…. = $-1.\overline{3}$. Example: The fraction $\frac{1}{4}$ has both the terminating decimal representation 0.25 and the repeating decimal expansion 0.24999999…. .

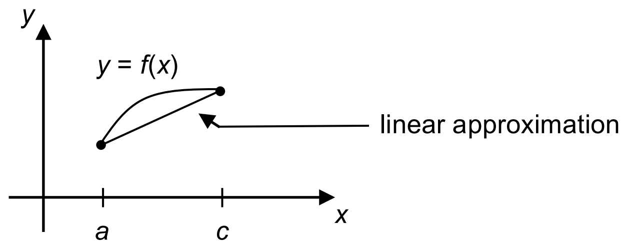
deductive reasoning	Deductive reasoning is a form of reasoning in which the conclusion is justified by an argument based on definitions, known facts, and accepted rules of logic. Example: A mathematical proof is a form of deductive reasoning.
degree measure	The degree measure of an angle is a measure of the size of the angle. See angle.
dependent	Two events are dependent if the occurrence (or nonoccurrence) of one event affects the probability of the other occurring. In an experimental context, two trials are dependent if the outcome of one affects the other. Example: In a probability experiment, there are two red marbles and one blue marble in a bag. One marble will be chosen randomly out of the bag, then another. The probability of getting a red marble on the second pick is dependent upon the result of the first pick. In the bag: red red blue → If red picked → left in bag: red blue → If blue picked → left in bag: red red
dependent variable	A dependent variable is a variable whose value is determined by the values of the independent variables. Example: For the function $y = 3x^2 + 1$, y is the dependent variable and x is the independent variable. When a value is assigned to x, the value of y is completely determined.
diameter	A diameter of a circle is a line segment joining two points of the circle that passes through the center of the circle. Example: The line segment from E to F is a diameter. 

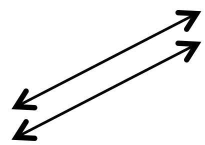
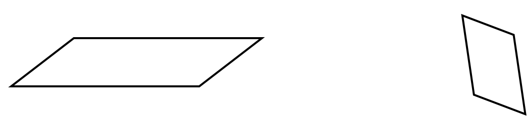
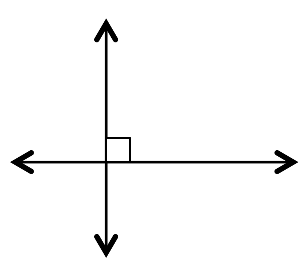
dilation	A dilation is a transformation that moves each point along the ray through the point emanating from a fixed center, multiplying distances from the center by a common scale factor. Example: The transformation of the plane mapping $(x, y) \rightarrow (2x, 2y)$ is a dilation with center at the origin and scale factor 2.
directed line segment	A directed line segment (or oriented line segment) is a line segment for which a direction from one endpoint to the other is specified. Example: The line segment with endpoints P and Q has two possible directions (orientations), from P to Q and from Q to P.
disk	A disk is the region in a plane bounded by a circle. It includes all points in the plane whose distance from a specified point (the center) is less than a fixed distance (the radius).
distance	The distance between two points on a number line is the absolute value of their difference. Example: The distance from 3 to -4 is $\left\|3 - (-4)\right\| = \left\|3 + 4\right\| = 7$.
empirical evidence	Empirical evidence is evidence that is based on experience or observation. Data collected from experimentation are called empirical data.
estimate	An estimate is an educated guess.
exponential notation	The exponential notation b^n (read as " b to the power n ") is used to express n factors of b. The number b is the base, and the natural number n is the exponent. Exponential notation is extended to arbitrary integer exponents by setting $b^0 = 1$ and $b^{-n} = \frac{1}{b^n}$. Example: $2^3 = 2 \cdot 2 \cdot 2 = 8$ (the base is 2 and the exponent is 3) Example: $3^2 \cdot 5^3 = 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 = 1{,}125$ (the bases are 3 and 5) Example: $2^0 = 1$ Example: $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

exterior angle of a triangle	An exterior angle of a triangle is an angle formed by a side of the triangle and an extension of its adjacent side.  Example: $\angle BCD$ is an exterior angle of ΔABC.
face	A face of a polyhedron is one of the polygons that are joined together to form the polyhedron. Example: A cube has six faces. 
factor	In a multiplication problem, a factor is a number or expression being multiplied. See product. Example: 3 • 5 = 15 factor factor product
fourfold way	The fourfold way refers to a collection of four ways to approach a math problem: • Numbers (numerical approach, as by a making a t-table) • Pictures (visual approach, as with a picture or graph) • Words (verbalizing a solution, orally or in writing) • Symbols (approaching the problem using algebraic symbols) Each approach may lead to a valid solution. Collectively they should lead to a complete and comprehensive solution, one that is readily accessible to more learners and that provides more insight.
frequency table	A frequency table is a table that lists items and the number of times they occur in a data set.
function	A function is a rule that assigns to each input value exactly one output value. A function may also be referred to as a transformation or mapping. The collection of output values is the image of the function. Example: Consider the function $y = 3x + 6$. For any input value, say $x = 10$, there is a unique output value, in this case $y = 36$. This output value is obtained by substituting the value of x into the equation. Example: The function $y = x^2 + 1$ assigns to the input value $x = 2$ the output value $y = 2^2 + 1 = 5$.

generalization	Generalization is the process of formulating general concepts by abstracting common properties from specific cases. Example: We may notice that the sum of two numbers is the same regardless of order (as in 3 + 5 = 5 + 3). We generalize by writing the statement: $a + b = b + a$ for all numbers a and b.
graph of a function	The graph of a function is the set of ordered pairs, each consisting of an input and its corresponding output. If the inputs and outputs are real numbers, then we can represent the graph of a function as points on the coordinate plane. Example: The graph of $y = 2x + 1$ on the coordinate plane. 
height	The height of a figure is the length of an altitude. See altitude of a triangle, altitude of a parallelogram.
hypotenuse	The hypotenuse of a right triangle is the side of the triangle opposite the right angle. It is the longest side in a right triangle. See right triangle. Example: 
image	The image of a function or transformation is the collection of its output values. See function, transformation.
independent variable	An independent variable is a variable whose value may be specified. Once specified, the values of the independent variables determine the values of the dependent variables. Example: For the function $y = 3x^2 + 1$, y is the dependent variable and x is the independent variable. We may assign a value to x. The value assigned to x determines the value of y.

inductive reasoning	Inductive reasoning is a form of reasoning in which the conclusion is supported by the evidence but is not proved. Example: After creating a table of sums of odd numbers $1 + 3 = 4$, $1 + 3 + 5 = 9$, $1 + 3 + 5 + 7 = 16$, etc., we may reason inductively that the sum of the first n odd numbers is n^2.
inverse operation	The inverse operation to a mathematical operation reverses the effect of the operation. Example: Addition and subtraction are inverse operations. Example: Multiplication and division are inverse operations.
irrational number	An irrational number is a real number whose decimal expansion is nonrepeating. The irrational numbers are the real numbers that are not rational. Example: $\sqrt{20}$, π, and 0.101001000100001... are irrational numbers.
isosceles triangle	An isosceles triangle is a triangle that has at least two sides of equal length. 
legs	The legs of a right triangle are the two sides of the triangle adjacent to the right angle. Example: 
length	The length of a curve is a measure of distance along the curve.
line of best fit	A line of best fit for a scatter plot is a straight line that best represents (in some sense) the data points in the scatter plot. 

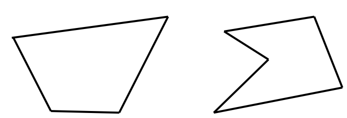
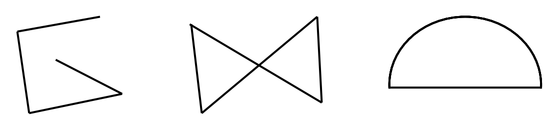
line segment	A line segment is a straight-line path joining two points. The line segment between two points P and Q, denoted by $\overline{PQ}$, consists of all points on the straight line through P and Q that lie between P and Q. The points P and Q are the endpoints of the line segment.
linear interpolation	Linear interpolation refers to a method of approximating the values of a function f at points of an interval $a < x < c$ by the values of the linear function that coincides with f at the endpoints a and c.  Example: We may approximate $\sqrt{x}$ for $x > 25$ and $x < 36$ by the linear function $y = 5 + \left(\frac{1}{11}\right)(x - 25)$, which has values $y = \sqrt{25} = 5$ at $x = 25$ and $y = \sqrt{36} = 6$ at $x = 36$.
measurement data	Measurement data is data that comes from making measurements. Example: Measurement data can be obtained by measuring such things as heights, weights, temperatures, lengths, areas, and volumes.
multiple	A multiple of a number m is a number of the form $k \cdot m$ for any integer k. Example: The numbers 5, 10, 15, and 20 are multiples of 5, because $1 \bullet 5 = 5$, $2 \bullet 5 = 10$, $3 \bullet 5 = 15$, and $4 \bullet 5 = 20$.
natural numbers	The natural numbers are the numbers 1, 2, 3, … . Natural numbers are also referred to as counting numbers.
numerical data	Numerical data is data consisting of numbers.
obtuse angle	An obtuse angle is an angle whose measure is between 90° and 180°. See angle.

outlier	An outlier of a data set is a data value that is unusually small or unusually large relative to the overall pattern of values in the data set. Example: For the data set {1, 1, 1, 3, 5, 6, 6, 7, 23}, the data value 23 is a potential outlier.
parallel	Two lines in a plane are parallel if they do not meet. Two line segments in a plane are parallel if the lines they lie on are parallel. 
parallelogram	A parallelogram is a quadrilateral in which opposite sides are parallel. In a parallelogram, opposite sides have equal length and opposite angles have equal measure. 
perfect square	A perfect square, or square number, is a number that is the square of a natural number. Example: The area of a square with integral side-length is a perfect square. The perfect squares are $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, $16 = 4^2$, $25 = 5^2$, … .
perimeter	The perimeter of a plane figure is the length of the boundary of the figure. Example: The perimeter of a square is four times its side-length. The perimeter of a rectangle is twice the length plus twice the width. The perimeter of a circular disk is its circumference, which is π times its diameter.
perpendicular	Two lines are perpendicular if they intersect at right angles. 

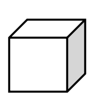
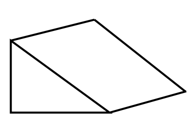
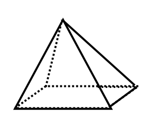
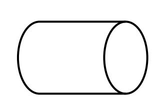
<table>
<tr><td>pi</td><td><u>Pi</u> (written π) is the Greek letter used to denote the value of the ratio of the circumference of a circle to its diameter. Pi is an irrational number, with decimal representation $\pi = 3.14159\ldots$. The rational numbers 3.14 and $\frac{22}{7}$ are often used to approximate π.</td></tr>
<tr><td>place value number system</td><td>A <u>place value number system</u> is a positional number system in which the value of a digit in a number is determined by its location or place.

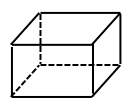
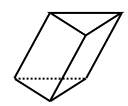
Example: In the number 7,863.21, the 8 is in the hundreds place and represents 800. The 1 is in the hundredths place and represents 0.01.

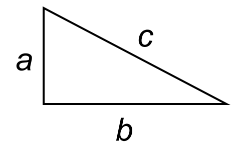
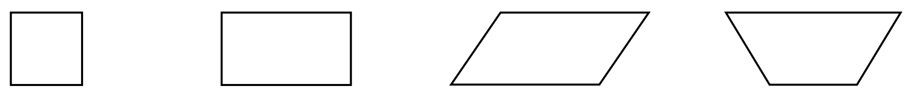
ten millions | millions | hundred thousands | ten thousands | thousands | hundreds | tens | ones | • | tenths | hundredths | thousandths | ten thousandths</td></tr>
<tr><td>polygon</td><td>A <u>polygon</u> is a simple closed curve in a plane consisting of a finite number of line segments joined end-to-end. The line segments are the <u>sides</u> (or <u>edges</u>) of the polygon, and the endpoints of the line segments are the <u>vertices</u> of the polygon.

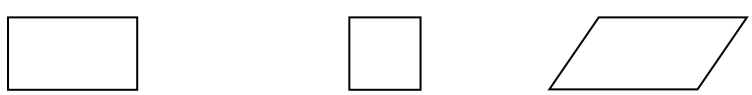

polygons

not polygons</td></tr>
<tr><td>polyhedron</td><td>A <u>polyhedron</u> is a closed figure in three-dimensional space consisting of a finite number of polygons that are joined at their edges and that form the boundary of the enclosed solid figure. The polygons are the <u>faces</u> of the polyhedron, the edges of the polygons are the <u>edges</u> of the polyhedron, and the vertices of the polygons are the <u>vertices</u> of the polyhedron

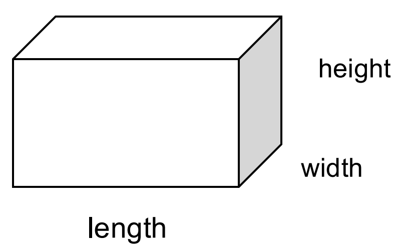
Example: A cube is a polyhedron. It has 6 faces, 12 edges, and 8 vertices. A cylinder is not a polyhedron.

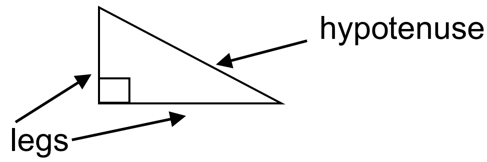

polyhedron (cube)

polyhedron (triangular prism)

polyhedron (square pyramid)

not a polyhedron (cylinder)</td></tr>
</table>

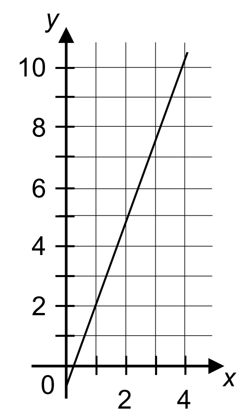
population	In statistics, the population refers to the source of a data set. Example: If we wish to make statistical inferences about the students at a school, we may take a random sample of the students, or we may gather data from all the students. In either case, the population refers to the students in the school.
prism	A prism is a polyhedron in which two faces (the bases) are congruent parallel polygons, and the other faces (the lateral faces) are parallelograms. If the lateral faces are perpendicular to the bases, the prism is a right prism. Otherwise, the prism is an oblique prism. Example: A right rectangular prism is a right prism whose bases are rectangles. Example: A triangular prism is a prism whose bases are triangles.  right rectangular prism  oblique triangular prism
product	A product is the result of multiplying two or more numbers or expressions. The numbers or expressions being multiplied to form the product are factors of the product. Example: The product of 7 and 8 is 56, written $7 \cdot 8 = 56$. The numbers 7 and 8 are both factors of 56.
proof	A proof of a mathematical statement is an argument based on definitions, previously established theorems, and accepted rules of logic to justify the statement. See deductive reasoning.
proportional	Two quantities are proportional if one is a multiple of the other. We say that y is proportional to x if $y = kx$, where k is the constant of proportionality. Example: In a scale drawing of a figure, lengths associated with the scale drawing are proportional to the corresponding lengths associated with the original figure. The constant of proportionality is the scale factor of the scale drawing.

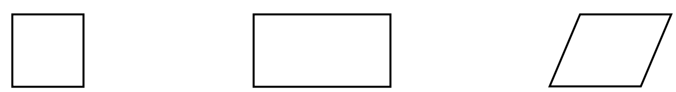
Pythagorean Theorem	The Pythagorean Theorem states that the sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse. See converse of the Pythagorean Theorem. $a^2 + b^2 = c^2$  Example: If the lengths of the legs of a right triangle are 5 and 12 units respectively, then the hypotenuse has length 13 units, because $13^2 = 5^2 + 12^2$.
quadrilateral	A quadrilateral is a polygon with four sides.  square rectangle parallelogram trapezoid
radical expression	A radical expression is an expression involving a root, such as a square root. Example: $\sqrt{20}$ and $5\sqrt{3}$ are radical expressions
radicand	In a radical expression, the radicand is the quantity or expression under the square root symbol or radical symbol. Example: In the expression $\sqrt{20}$, the radicand is 20.
radius	The radius of a circle (or sphere) is the distance from a point on the circle (or sphere) to its center. A line segment with one endpoint lying on the circle (or sphere) and the other at its center is also referred to as a radius of the circle (or sphere).

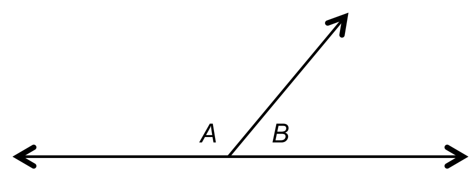
rational numbers	Rational numbers are quotients of integers. Rational numbers can be expressed as $\frac{m}{n}$, where m and n are integers and $n \neq 0$. Example: $\frac{3}{5}$ is rational because it *is* a quotient of integers. Example: $2\frac{1}{3}$ and 0.7 are rational numbers because they *can be* expressed as quotients of integers ($2\frac{1}{3} = \frac{7}{3}$ and $0.7 = \frac{7}{10}$). Example: $\sqrt{2}$ and π are NOT rational numbers. They *cannot be* expressed as a quotient of integers.
real numbers	The real numbers form an arithmetic system with operations of addition and multiplication, which has an ordering, and in which one can find limits of certain sequences. Example: The real numbers can be viewed as the set of all decimals (and their negatives), with the convention that a terminating decimal (such as 4.2) is regarded as the same real number as the repeating decimal obtained by replacing the final digit by one less followed by all 9's (in this case 4.19999….). Example: The real numbers can also be visualized geometrically as the real number line, a straight line on which the real numbers serve as coordinates.
rectangle	A rectangle is a quadrilateral with four right angles. In a rectangle, opposite sides are parallel and have equal length. Example: A square is a rectangle with four congruent sides.  rectangle square not a rectangle

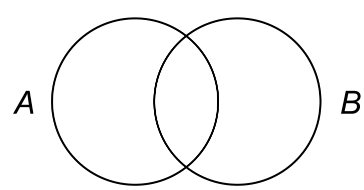
reflection	The reflection of a plane through a line L is the transformation that maps each point to its mirror image on the other side of L, that is, that maps P to the point P' such that L is the perpendicular bisector of the line segment PP' from P to P'. The line L is called the line of reflection. Example: The transformation $(x, y) \rightarrow (x, -y)$ mapping $P = (x, y)$ to $P' = (x, -y)$ is a reflection of the plane through the x-axis.
relative frequency table	A relative frequency table is a frequency table that lists items and the proportion (or percent) of times they occur.
repeating decimal	A repeating decimal is a decimal that ends in repetitions of the same block of digits. A terminating decimal is regarded as a repeating decimal that ends in all zeros. Example: The repeating decimal 52.19343434… ends in repetitions of the block “34.” An abbreviated notation for the decimal is $52.19\overline{34}$, where the bar over 34 indicates that the block is repeated. Example: The terminating decimal 4.62 is regarded as a repeating decimal. Its value is 4.620000… .
right angle	A right angle is an angle that measures 90°. See angle.
right rectangular prism	A right rectangular prism (or box) is a six-sided polyhedron in which all the faces are rectangles. The opposite faces of a right rectangular prism are parallel to each other. The distances between pairs of opposite faces are the length, width, and height of the right rectangular prism. 

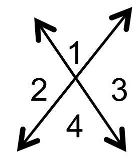
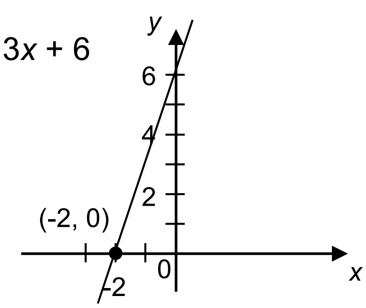
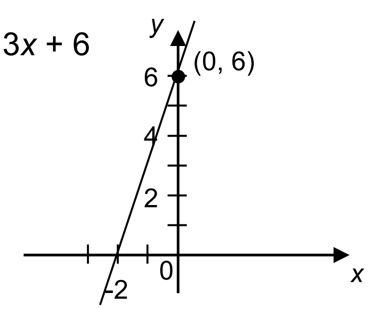
right triangle	A right triangle is a triangle that has a right angle. The two sides of the triangle adjacent to the right angle are the legs of the triangle, and the side opposite the right angle is the hypotenuse of the triangle. 
rigid motion	A rigid motion is a transformation that preserves distances. Any rigid motion of the plane is a sequence of one or more translations, rotations, and reflections. Rigid motions also preserve lengths and angle measures. Example: Translations, rotations, and reflections of the plane are rigid motions, that is, when P is mapped to P' and Q is mapped to Q' by such a transformation, the distance $\lvert PP' \rvert$ from P to P' is equal to the distance $\lvert QQ' \rvert$ from Q *to* Q'.
rotation	A rotation of a plane is a transformation that turns it through a given angle about a given point. The given angle is called the angle of rotation, and the given point is called the center point of rotation. Example: The transformation $(x,y) \rightarrow (-y,x)$ is a rotation of the plane about the origin through angle 90°.
rounding	Rounding refers to replacing a number by a nearby number that is easier to work with or that better reflects the precision of the data. Typically, rounding requires the changing of digits to zeros. Example: 15,632 rounded to the nearest thousand is 16,000. Example: 12.83 rounded to the nearest tenth is 12.80, or 12.8.
scale factor	A scale factor is a positive number which multiplies some quantity.
scientific notation	Scientific notation for a positive number represents the number as a product of a decimal between 1 and 10 and a power of 10. It is typically used to write either very large numbers or very small numbers. Example: In scientific notation, the number 245,000 is written as 2.45×10^5. Example: In scientific notation, the number 0.0023 is written as 2.3×10^{-3}.

simplify	Simplify refers to converting a numerical or variable expression to a simpler form. A variable expression might be simplified by combining like terms. A fraction might be simplified by dividing numerator and denominator by a common divisor. Example: $2x + 6 + 5x + 3 = 7x + 9$ Example: $\frac{8}{12} = \frac{2}{3}$
slope of a line	The slope of a line is the vertical change (change in the y-value) per unit of horizontal change (change in the x-value). The slope is sometimes described as the ratio of the "rise to the run." Example: The slope of the line through (1,2) and (4,10) is $\frac{8}{3}$: $\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{(\text{difference in } y)}{(\text{difference in } x)} = \frac{(10-2)}{(4-1)} = \frac{8}{3}$ 
slope-intercept form	The slope-intercept form of the equation of a line is the equation $y = mx + b$, where m is the slope of the line, and b is the y-intercept of the line. Example: The equation y = 2x + 3 determines a line with slope 2 and y-intercept 3.
solution set	A solution set is the set of values that satisfy a given set of equations or inequalities.
solution to an equation	A solution to an equation involving variables consists of values for the variables which, when substituted, make the equation true. Example: The value $x = 8$ is a solution to the equation $10 + x = 18$. If we substitute 8 for x in the equation, the equation becomes true: $10 + 8 = 18$.

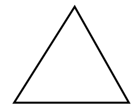
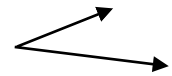
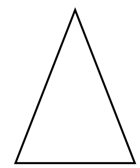
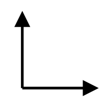
solve an equation	<u>Solve an equation</u> refers to finding all values for the variables in the equation that, when substituted, make the equation true. Values that make an equation true are called solutions to the equation. Example: To solve the equation $2x = 6$, one might think "two times what number is equal to 6?" The only value for x that satisfies this condition is 3, which is then the solution. Example: Solve the equation $2x + 6 = 4x$. $\begin{aligned} 2x + 6 &= 4x \\ -2x \quad &= -2x \\ 6 &= 2x \\ 3 &= x \end{aligned}$ 3 is the solution to the equation.
sphere	A <u>sphere</u> is a closed surface in three-dimensional space consisting of all points at a fixed distance (the radius) from a specified point (the center).
square	A <u>square</u> is a rectangle whose sides have equal length.  is a square not a square not a square
square of a number	The <u>square of a number</u> is the product of the number with itself. Example: The square of 5 is 25, since 52 = (5)(5) = 25. The square of -5 is also 25, since (-52) = (-5)(-5) = 25.
square root	A <u>square root</u> of a number n is a number whose square is equal to n, that is, a solution of the equation $x^2 = n$. The positive square root of a number n, written $\sqrt{n}$, is the positive number whose square is n. The symbol $\sqrt{}$ is called a radical sign, and the number under the radical sign is called the radicand. Except where otherwise noted, the term "the square root of n" refers to the positive square root. Example: Both 5 and -5 are square roots of 25, because $5^2 = 25$ and $(-5)^2 = 25$. The positive square root of 25 is 5.

substitution	Substitution refers to replacing a value or quantity with an equivalent value or quantity. Example: If $x + y = 10$, and we know that $y = 8$, then we may substitute this value for y in the equation to get $x + 8 = 10$.
supplementary angles	Two angles are supplementary if the sum of their measures is 180°. Example: Any two right angles are supplementary, because the sum of their measures is $90° + 90° = 180°$. Example: Angles A and B below are supplementary because they determine a straight line, or 180°. 
symmetric property of equality	The symmetric property of equality states that if $a = b$, then $b = a$.
system of linear equations	A system of linear equations is a set of two or more linear equations in the same variables.
term	The terms in a mathematical expression involving addition (or subtraction) are the quantities being added (or subtracted). Terms that have the same variable part are referred to as like terms. Example: Expression: $2x + 6 + 3x + 6$ Terms: $2x$, 6, $3x$, and 5 Like terms: The terms $2x$ and $3x$ are like terms because they have the same variable part (x). The terms 6 and 5 are like terms because they are both constants (or their variable part can be considered as x^0).
terminating decimal	A terminating decimal is a decimal whose digits are 0 from some point on. The final 0's in the expression for a terminating decimal are usually omitted. Example: 4.62 is a terminating decimal with value $4 + \frac{6}{10} + \frac{2}{100}$.

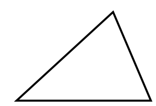
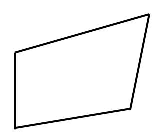
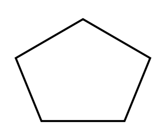
transformation	A <u>transformation</u> is a function. The image of the transformation is the collection of values of the function. A transformation of a plane is a mapping of the plane to itself, that is, a function defined on the plane whose image is a subset of the plane. See <u>function</u>. Example: Translations, rotations, reflections, and dilations are transformations of the plane.
transitive property of equality	The <u>transitive property of equality</u> states that if $a = b$ and if $b = c$, then $a = c$.
translation	The <u>translation</u> of the plane by P is the transformation of the plane that maps a point A to $A' = A + P$. Example: The transformation $T(x, y) = (x + 1, y + 2) = (x, y) + (1, 2)$ is a translation. It slides all points of the plane the same distance, along lines parallel to the line through the origin and (1, 2).
trend line	A <u>trend line</u> refers to a line that is suggested by a collection of points graphed on a coordinate grid.
variable	A <u>variable</u> is a quantity whose value has not been specified. Variables are used in many different ways. They may refer to functions, to quantities that vary in a relationship, or to unknown quantities in equations and inequalities. Example: In the equation $d = r t$, the quantities d, r, and t are variables. In the equation $2x + 6 = 10$, the variable x may be referred to as the unknown.
Venn diagram	A <u>Venn diagram</u> is a pictorial way to represent relationships between sets, in which the sets are represented by regions in the plane. Example: 

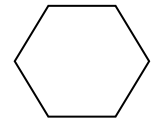
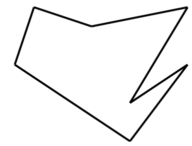
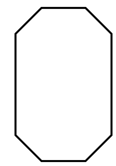
vertical angles	Two angles are <u>vertical angles</u> if they are the opposite angles formed by a pair of intersecting lines. When two lines intersect at a point, they form two pairs of vertical angles with vertex at the point. Example: $\angle 1$ and $\angle 4$ are vertical angles. Example: $\angle 2$ and $\angle 3$ are vertical angles. 
vertical line test	The <u>vertical line test</u> asserts that a subset of the plane is the graph of a function if and only if each vertical line in the plane meets the set in at most one point. In particular, if a vertical line intersects a set in the plane in more than one point, the set cannot be the graph of a function.
whole numbers	The <u>whole numbers</u> are the natural numbers together with 0. They are the numbers 0, 1, 2, 3, ….
x-intercept	The <u>*x*-intercept</u> of a line is the *x*-coordinate of the point at which the line crosses the *x*-axis. It is the value of x that corresponds to $y = 0$. Example: The *x*-intercept of the line $y = 3x + 6$ is -2. If $y = 0$, then $x = -2$. 
y-intercept	The <u>*y*-intercept</u> of a line is the *y*-coordinate of the point at which the line crosses the *y*-axis. It is the value of y that corresponds to $x = 0$. Example: The *y*-intercept of the line $y = 3x + 6$ is 6. If $x = 0$, then $y = 6$. 

GREEK AND LATIN WORD ROOTS

equi	"equal": An equilateral triangle has three equal sides. The sides of an equilateral polygon have the same length. 
gon	"angle": A pentagon has five angles. 
hedron	"seat," "base"; refers to faces of a solid figure. A decahedron is a polyhedron with ten faces.
iso	"equal," "the same": An isosceles triangle has two equal sides. Isomorphic objects have the same form. Some folks maintain their equilibrium by doing isometric exercises. "equi" comes from Latin, while "iso" comes from Greek. 
lateral	"side": A quadrilateral has four sides. A movement sideways is a lateral movement.
ortho	"right angle," "upright," "correct": Two lines are orthogonal if they are perpendicular. Orthorhombic crystals have three axes that come together at right angles. An orthodontist corrects teeth. An orthodox person seeks to be correct. 
peri	"around," "about": The perimeter of a plane figure is the distance around it. A periscope allows one to see around an obstruction. Paul Erdös was a well-known peripatetic mathematician.
poly	"many": A polygon has many (three or more) angles. A polygraph records several body activities simultaneously. A polygamist has more than one spouse. Los Angeles schools have a polyglot student body.

NUMBER PREFIXES

uni	"one": The units place is to the left of the decimal. We form the union of sets by unifying them. Some problems have unique solutions. A unicycle has one wheel. A unicorn has one horn.
mono	"alone," "one": A monomial is a polynomial with only one term. A monorail has one rail. He delivered his monologue about monogamous relationships in a dull monotone.
bi, bis	"two": The binary number system has base two. We bisect an angle. Property taxes are collected biannually, in March and November. Congressional elections are held biennially, in even-numbered years. Man and ape are bipeds.
di, dy	"two": The angle between two planes is a dihedral angle. A dyadic rational number is the quotient of an integer and a power of two. Physicists deal with dipoles, chemists with dioxides. A diphthong is a gliding speech form with two sounds. "di" comes from Greek, "bi" from Latin.
tri	"three": A triangle has three sides. A triathlon has three events: cycling, swimming, and running. 
quad	"four": A quadrilateral is a polygon with four sides. Coordinate axes divide the plane into four quadrants. The Olympic Games are a quadrennial event. 
tetra, tra	"four": A tetrahedron is a solid figure with four faces. A trapeze is a flying quadrilateral. "tetra" stems from Greek, while "quad" stems from Latin. The "tra" in "trapezoid" and "trapeze" is an abbreviated form of "tetra."
penta	"five": A pentagon has five sides. A pentahedron is a solid figure with five faces. The Pentagon Building in Washington, D.C., has five sides. 

hexa	"six": A hexagon has six sides. The cells in a beehive are hexagonal. 
hepta	"seven": A heptagon has seven sides. A heptahedron has seven faces. 
octa	"eight": An octagon has eight sides. The three coordinate planes divide space into eight octants. The musical octave has eight whole notes. An octopus has eight tentacles. 
nona	"nine": A nonagon has nine sides. A nonagenarian is in his or her nineties. Another (rarely used) word for a nine-sided polygon is "enneagon." The prefix "nona" comes from Latin, "ennea" from Greek.
deca	"ten": A decagon has ten sides. A decade has ten years. A decathlon has ten track-and-field events.
dodeca	"twelve": A dodecahedron has twelve faces. The regular dodecahedron is one of the five Platonic solids. It is occasionally used for calendars.
icosa	"twenty": An icosahedron has twenty faces. The regular icosahedron is one of the five Platonic solids. It has twelve vertices, and it can be obtained from a regular dodecahedron by placing a vertex at the center of each face of the dodecahedron.
cent	"hundred": A centipede has a hundred legs. A centimeter is a hundredth of a meter. The St. Louis fair celebrated the centennial of the Lewis and Clark expedition.
milli	"thousand": We have entered a new millennium. A millisecond is a thousandth of a second.

EXPONENTS AND ROOTS

Square Roots: Estimates Versus Exact Value

Q: What is the square root of 9?

A: We know that $3^2 = 9$, so $\sqrt{9} = 3$.

Q: What is the square root of 7?

A: We know of no rational number that, when squared, is equal to 7.

Using the square root function on a simple calculator, we get an approximation to several decimal places: $\sqrt{7} \approx 2.645751$.

But by multiplication: $(2.645751)^2 = (2.645751)(2.645751) = 6.999998354$.

Find another approximation for $\sqrt{7}$ using a calculator with greater capacity, then square that number, and it will still not be exactly equal to 7.

We know that $\sqrt{7}$ is an irrational number (this is a packet 16 topic) and its decimal expansion is infinite with no block of digits that repeats.

So how do we write $\sqrt{7}$? The only way to write it exactly is to leave it in square root form.

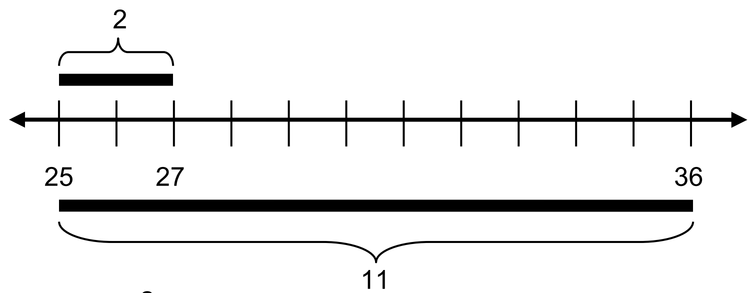
If we choose to approximate $\sqrt{7}$, the simplest way may be to state which two consecutive integers it is between. We know that $2^2 = 4$ and $3^2 = 9$, and we also know that 7 is between 4 and 9, so $\sqrt{7}$ is between 2 and 3.

Estimating Square Roots With Greater Accuracy

The following two strategies may be helpful for square root estimation. Note: the method illustrated below is referred to as <u>linear interpolation</u>.

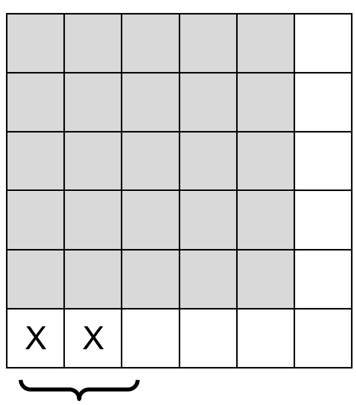
Find an estimate of $\sqrt{27}$ as a mixed number and as a whole number with decimal remainder.

<u>Strategy 1</u>: Since $\sqrt{27}$ is between $\sqrt{25}$ and $\sqrt{36}$ (25 and 36 are perfect squares), provide a "magnified" portion of a number line from 25 to 36.



The distance from 25 to 27 is $\frac{2}{11}$ of the distance from 25 to 36. Hence, the distance from $\sqrt{25}$ to $\sqrt{27}$ should be about $\frac{2}{11}$ of the distance from $\sqrt{25}$ to $\sqrt{36}$. In other words, the distance from 5 to $\sqrt{27}$ should be about $\frac{2}{11}$ of the distance from 5 to 6. Thus $\sqrt{27} \approx 5\frac{2}{11}$.

<u>Strategy 2</u>: Since 25 and 36 are perfect squares, on grid paper, draw a 5 x 5 square inside a 6 x 6 square as illustrated below.



The two X's represent the 26th and 27th unit squares.

- The larger square is $6 \times 6 = 6^2 = 36$ unit squares.
- The smaller, shaded square inside is $5 \times 5 = 5^2 = 25$ unit squares.

So, $\frac{27 - 25}{36 - 25} = \frac{2}{11}$

Therefore, $\sqrt{27}$ is about $5\frac{2}{11}$. (calculator check: $5\frac{2}{11} = 5.\overline{18}$ and $\sqrt{27} \approx 5.196$)

Numbers Squared and Cubed

Why do we say that a number raised to the second power is "squared"? The reason has to do with the area formula for squares. The area of a square of side length s is given by

$$\text{area} = s \bullet s = s^2.$$

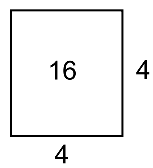
A square with side length 4 units has area "4 squared" = 4^2 = 16 square units.

What about "square root" – where does that term come from?

Here the reason is that a "root" can also refer to the solution of an equation. A "square root" has to do with finding the side length of a square of a given area; that is, of solving the equation $s^2 = A$. For a given area A, the side length s of the square with area A is

side length = $s = \sqrt{A}$ = "square root of A."

A square with area 16 square units has side length $\sqrt{16} = 4$ units.



$\rightarrow$ $4^2 = 64$ and $\sqrt{16} = 4$

Why do we say that a number raised to the third power is "cubed"? In this case, the answer has to do with the volume formula for cubes. The volume of a cube with side length s is given by

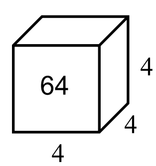
$$\text{volume} = s \cdot s \cdot s = s^3.$$

A cube with side length 4 units has volume "4 cubed" = 4^3 = 64 cubic units.

In turn, a "cube root" has to do with finding the side length of a cube of a given volume, that is, of solving the equation $s^3 = V$. For a given volume V, the side length s of the cube with volume V is

side length = $s = \sqrt[3]{V}$ = "cube root of V."

A cube with volume 64 cubic units has side length $\sqrt[3]{64}$ = 4 units.



$\rightarrow$ $4^2 = 64$ and $\sqrt[3]{64} = 4$

Although we assume here that V is positive, the cube root of a negative number can be found by solving the equation, $s^3 = V$. The square root of a negative number is not a real number.

Squaring a number and finding the square root of a number are inverse operations. Similarly, cubing a number and finding the cube root of a number are inverse operations.

Some Rules for Exponents

- Product Rule for exponentials: $x^a \bullet x^b = x^{a+b}$

  Example: $3^2 \bullet 3^4 = (3 \bullet 3)(3 \bullet 3 \bullet 3 \bullet 3) = 3^6 = 3^{2+4}$

- Power Rule for exponentials: $(x^a)^b = x^{a \bullet b}$

  Example: $(3^2)^4 = (3^2)(3^2)(3^2)(3^2) = (3 \bullet 3)(3 \bullet 3)(3 \bullet 3)(3 \bullet 3) = 3^8 = 3^{2 \bullet 4}$

- Quotient Rule for exponentials: $\dfrac{x^a}{x^b} = x^{a-b}$

  Example 1: $\dfrac{3^5}{3^2} = \dfrac{3 \bullet 3 \bullet 3 \bullet 3 \bullet 3}{3 \bullet 3} = \dfrac{3 \bullet 3 \bullet 3}{1} = 3 \bullet 3 \bullet 3 = 3^3 = 3^{5-2}$

  Example 2: $\dfrac{3^2}{3^5} = \dfrac{3 \bullet 3}{3 \bullet 3 \bullet 3 \bullet 3 \bullet 3} = \dfrac{1}{3 \bullet 3 \bullet 3} = \dfrac{1}{3^3} = 3^{-3} = 3^{2-5}$

  Example 3: $\dfrac{3^2}{3^2} = \dfrac{3 \bullet 3}{3 \bullet 3} = 1 = \;\; = 3^0 = 3^{2-2}$

- **Heads Up!** These rules apply to expressions with the same base numbers.

  Example: $2^4 \bullet 3^2 = (2 \bullet 2 \bullet 2 \bullet 2) \bullet (3 \bullet 3)$, and the product rule does not apply.

Zero and Negative Exponents				
These patterns show that the rules for zero and negative exponents are reasonable.				
Pattern: Divide by 2		**Result**	**Pattern as a product**	**Pattern in exponential form**
		Start with 8	$2 \bullet 2 \bullet 2$	2^3
$8 \div 2$	=	4	$2 \bullet 2$	2^2
$4 \div 2$	=	2	2	2^1
$2 \div 2$	=	1	1	2^0
$1 \div 2$	=	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2^1}$ or 2^{-1}
$\frac{1}{2} \div 2$	=	$\frac{1}{4}$	$\frac{1}{2 \bullet 2}$	$\frac{1}{2^2}$ or 2^{-2}
$\frac{1}{4} \div 2$	=	$\frac{1}{8}$	$\frac{1}{2 \bullet 2 \bullet 2}$	$\frac{1}{2^3}$ or 2^{-3}

Rules for Zero and Negative Exponents	
Rule 1: $x^0 = 1 \ (x \neq 0)$	Rule 2: $x^{-a} = \frac{1}{x^a}$
Example: $5^0 = 1$	Example: $10^{-1} = \frac{1}{10}$, $\frac{1}{9} = \frac{1}{3^2} = 3^{-2}$

Place Value Names

In the base-10 number system, each place has a value ten times that of the place value to its right and one-tenth the value of the place to its left.

thousands	hundreds	tens	ones	tenths	hundredths	thousandths	ten thousandths

To determine what a digit stands for in a number, multiply the digit by its place value. For example, in the numeral 345.67, the 4 stands for 4(10) = 40, and 7 stands for $7\left(\frac{1}{100}\right) = 0.07$.

Another way to interpret the meaning of a digit to the right of the decimal is to divide by the whole number part of the place value name. For example, in the numeral 345.67, the meaning of the 7 is $7 \div 100 = \frac{7}{100}$.

Scientific Notation

Given number	Related decimal between 1 and 10	Power of 10	Number in scientific notation	Reasoning
120,000,000	1.2	10^8	1.2×10^8	The number given is 10^8 times as large as 1.2; adjust place values by multiplication.
0.0000345	3.45	10^{-5}	3.45×10^{-5}	3.45 is 10^5 times as large as the given number; adjust place values by multiplication.

RATIONAL AND IRRATIONAL NUMBERS

Repeating decimals

A repeating decimal is a decimal that ends with repetitions of the same pattern of digits.

(A "repeat bar" can be placed above the digits that repeat.) Using a calculator or the long division algorithm are two ways to change these fractions to decimals.

Examples: $\frac{2}{9} = 0.222222 = 0.\overline{2}$; $\frac{2}{11} = 0.181818 = 0.\overline{18}$; $\frac{1}{2} = 0.500000 = 0.5\overline{0}$

A terminating decimal is a decimal whose digits are 0 from some point on. The final 0's in the expression for a terminating decimal are usually omitted. Even though the final 0's are omitted, a terminating decimal is regarded as a repeating decimal that ends in all zeros.

Examples: $\frac{1}{2}$ = 0.5000… = 0.5, $\frac{3}{4}$ = 0.75000… = 0.75

The repeating decimals above for $\frac{2}{9}$ and $\frac{2}{11}$ do not terminate. The other repeating decimals above terminate.

Rational Numbers

A rational number is number that *can be* written as a quotient $\frac{m}{n}$ of integers *m* and *n*, $n \neq 0$. Each of the following numbers is rational because it *can be* written in the above form.

$4 = \frac{4}{1} = \frac{8}{2}$ etc.	$-4 = \frac{-4}{1} = \frac{4}{-1}$ etc.	$0.3 = \frac{3}{10}$	$0.37 = \frac{37}{100}$
$-0.73 = \frac{-73}{100}$	$4.5 = 4\frac{5}{10} = \frac{45}{10}$	$7\frac{1}{3} = \frac{22}{3}$	$-5\frac{3}{4} = -\frac{23}{4}$

A Clever Procedure

Any repeating decimal can be written as a quotient of integers. Therefore, all repeating decimals are rational. The following algebraic idea is used to change a repeating decimal to a quotient of integers.

Example 1: Change $0.1\overline{6} = 0.166666\ldots$.
to a quotient of integers.

$$\begin{array}{rll} 10x = & 1.66666\ldots & (2) \\ \text{Let } \boldsymbol{x} = & \mathbf{0.16666\ldots} & \mathbf{(1)} \\ \hline 9x = & 1.5 & (3) \\ x = & \dfrac{1.5}{9} = \dfrac{15}{90} = \dfrac{1}{6} & (4) \end{array}$$

- Notice that step 2 is above step 1.
- The "trick" is to multiply both sides of the equation in step 1 by a power of 10 that will "line up" the repeating portion of the decimal.
- Subtract the expressions in step 1 from step 2. This results in a terminating decimal (step 3). Solve for x and simplify your result as a quotient of integers (step 4).

Example 2:

change $0.\overline{7} = 0.777777\ldots$
to a quotient of integers.

$$\begin{array}{rll} 10x = & 7.777777\ldots & (2) \\ \text{Let } \boldsymbol{x} = & \mathbf{0.7777777\ldots} & \mathbf{(1)} \\ 9x = & 7.000000\ldots & (3) \\ x = & \dfrac{7}{9} & (4) \end{array}$$

Example 3:

change $0.\overline{45} = 0.45454545\ldots$
to a quotient of integers.

$$\begin{array}{rll} 10x = & 45.454545\ldots & (2) \\ \text{Let } \boldsymbol{x} = & \mathbf{0.45454545\ldots} & \mathbf{(1)} \\ 99x = & 45.00000\ldots & (3) \\ x = & \dfrac{45}{99} = \dfrac{15}{33} = \dfrac{5}{11} & (4) \end{array}$$

Irrational Numbers
An irrational number is number that *can NOT be* written as a quotient of integers (cannot be written in the form $\frac{m}{n}$, $n \neq 0$). It is a number whose decimal expansion is not repeating. Every point on the number line that does not represent a rational number represents an irrational number. Examples include square roots of non-perfect squares like $\sqrt{2}$, $\sqrt{3}$, $\sqrt{11}$, and $\sqrt{33}$. A well-known irrational number is pi (π). None of these numbers can be written as quotients of integers. Also, their decimal expansions continue infinitely in a non repeating fashion.

Subsets of the Real Number System

Here are the standard notations for the most important subsets of the set of real numbers.

Natural numbers: also referred to as the Counting Numbers; 1, 2, 3, …

Whole numbers: the natural numbers together with zero; 0, 1, 2, 3, …

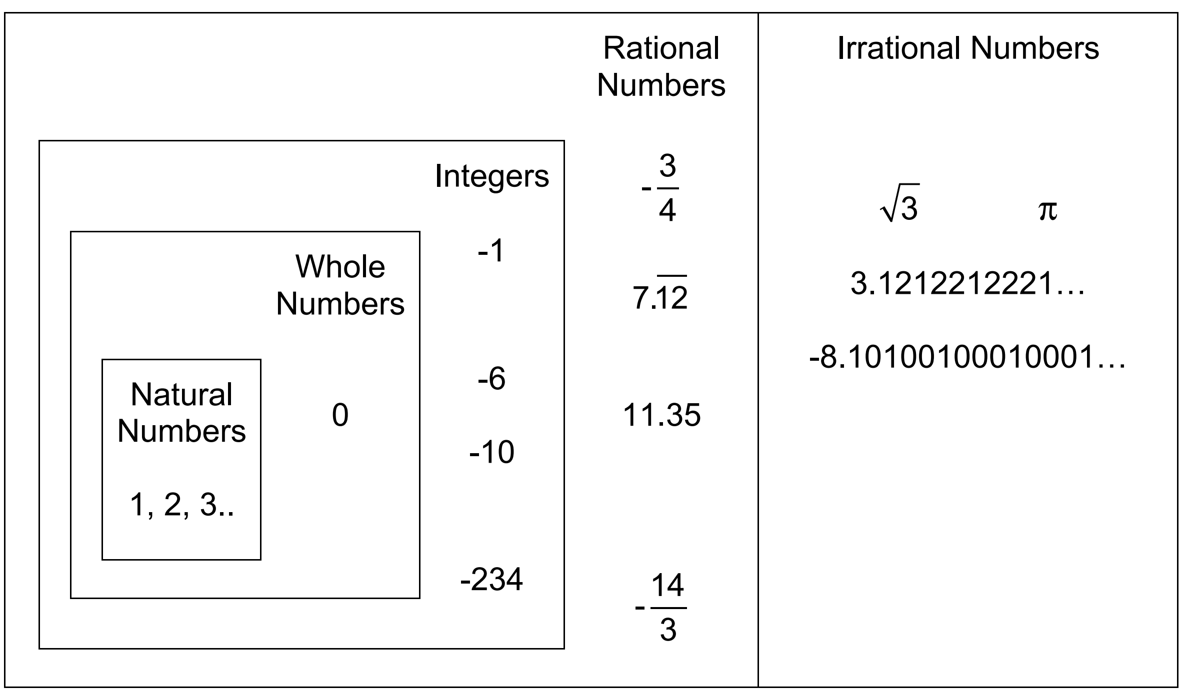
Integers: whole numbers and their opposites; …-3, -2, -1, 0, 1, 2, 3, …

Rational numbers: numbers that can be expressed as $\frac{m}{n}$, where m and n are integers and $n \neq 0$. These are numbers with repeating decimal expansions.

Irrational numbers: numbers that are not rational; their decimal expansions do not repeat.

Real numbers: they are the rational numbers together with the irrational numbers. Real numbers may be defined as finite or infinite decimals. Rational numbers correspond to repeating decimals, and irrational numbers correspond to nonrepeating decimals. The real numbers are visualized as a number line, on which every number has a decimal name (address).

Real Numbers



POINTS, LINES, AND ANGLES

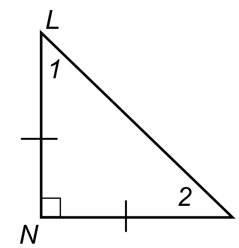
Geometry Notation

Here are some geometry notations used in these lessons.

Points are named by capital letters. The symbol for triangle is Δ. The symbol for angle is $\angle$.

The line segment from L to N is denoted by $\overline{LN}$.

Absolute value signs are used to denote nonnegative quantities that measure the "size" of something, as "length" or "angle measure."



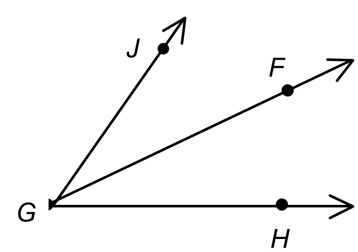
We denote the length of the line segment $\overline{LN}$ from L to N by $\left|\overline{LN}\right|$, or simply $|LN|$.

The single hash marks on the segments $\overline{LN}$ and $\overline{NM}$ indicate that the segments have equal length, that is, $\left|\overline{LN}\right| = \left|\overline{NM}\right|$.

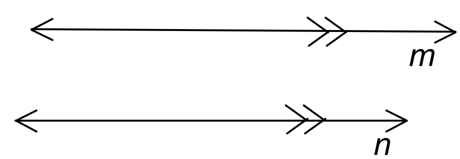
The measure of an angle $\angle N$ is denoted by $|\angle N|$. The small square at N indicates that $\angle LNM$ is a right angle, that is, that $|\angle LNM| = 90°$.

In naming a triangle, vertices may be listed in either a clockwise or counterclockwise direction. For example, the triangle may be named ΔLMN or ΔLNM.

In naming an angle, vertices may be listed in either a clockwise or counterclockwise direction. In the triangle above, the angle at the top can be denoted by $\angle NLM$, $\angle MLN$, $\angle L$ or simply $\angle 1$.



The pair of adjacent angles to the right are $\angle FGJ$ and $\angle HGF$. They share the common ray $\overrightarrow{GF}$. The two adjacent angles together form the angle $\angle JGH$.

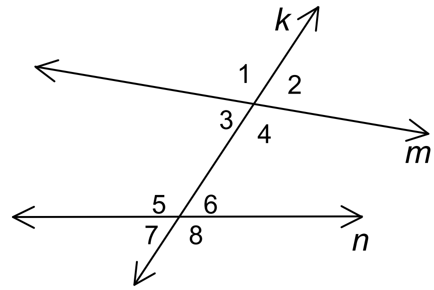


The double arrows on the lines m and n indicate that they are parallel, that is $m \parallel n$.

Transversals and Parallel Lines

A transversal is a line that passes through two or more other lines. In this diagram, line k is a transversal.

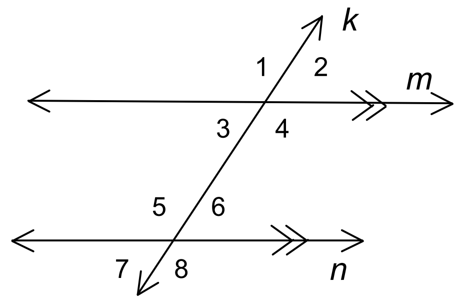
When two lines in a plane are cut (crossed) at two points by a transversal, eight angles are created. Some of these pairs of angles have special names.



Name of angle pairs	Examples	
alternate interior angles	$\angle 3$ and $\angle 6$	$\angle 4$ and $\angle 5$
alternate exterior angles	$\angle 1$ and $\angle 8$	$\angle 2$ and $\angle 7$
corresponding angles	$\angle 1$ and $\angle 5$ $\angle 3$ and $\angle 7$	$\angle 2$ and $\angle 6$ $\angle 4$ and $\angle 8$

Here are three important properties of the angles formed when a transversal cuts two parallel lines.

Line m is parallel to line n.
Line k is a transversal.



1. If two parallel lines are cut by a transversal, then alternate interior angles have the same measure.

   Examples: $|\angle 3| = |\angle 6|$ and $|\angle 4| = |\angle 5|$

2. If two parallel lines are cut by a transversal, then alternate exterior angles have the same measure.

   Examples: $|\angle 1| = |\angle 8|$ and $|\angle 2| = |\angle 7|$

3. If two parallel lines are cut by a transversal, then corresponding angles have the same measure.

   Examples: $|\angle 2| = |\angle 6|$ and $|\angle 4| = |\angle 8|$

Angle Sums in Triangles

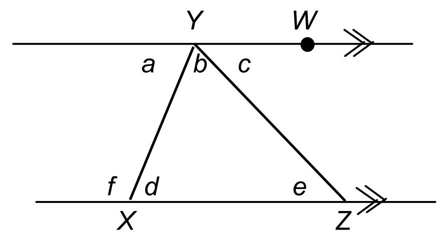
Here are two important facts about angle sums in triangles. They can be proved based on the properties of angles formed when a transversal cuts two parallel lines.

The sum of the measures of the angles in a triangle is equal to 180 degrees.

Symbolically: $|\angle b| + |\angle d| + |\angle e| = 180^\circ$

2. The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

Symbolically: $|\angle b| + |\angle e| = |\angle f|$



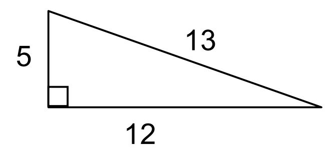
Statements	Reasons
There is a line $\overleftrightarrow{WY}$ passing through Y and parallel to $\overleftrightarrow{XZ}$.	Parallel postulate
$\lvert\angle a\rvert + \lvert\angle b\rvert + \lvert\angle c\rvert = 180^\circ$	Sum of measures of angles on a straight line equals 180°.
$\lvert\angle a\rvert = \lvert\angle d\rvert$ and $\lvert\angle c\rvert = \lvert\angle e\rvert$	If two lines are parallel, then alternate interior angles have equal measures.
Fact 1: $\lvert\angle b\rvert + \lvert\angle d\rvert + \lvert\angle e\rvert = 180^\circ$	Substitution.
$180^\circ = \lvert\angle f\rvert + \lvert\angle d\rvert$	The sum of the measures of supplementary angles is 180 degrees.
$\lvert\angle b\rvert + \lvert\angle d\rvert + \lvert\angle e\rvert = \lvert\angle f\rvert + \lvert\angle d\rvert$	Substitution.
Fact 2: $\lvert\angle b\rvert + \lvert\angle e\rvert = \lvert\angle f\rvert$	Addition property of equality (subtract).

PYTHAGOREAN THEOREM

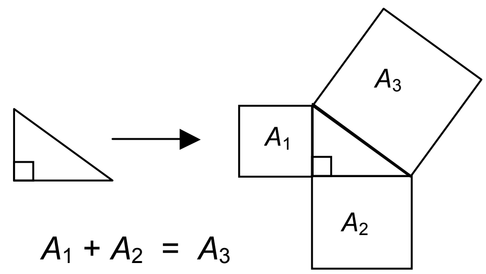
The Pythagorean Theorem

The Pythagorean Theorem states that the sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse.

Example: If the lengths of the legs of a right triangle are 5 and 12 units respectively, then the hypotenuse has length 13 units, because $5^2 + 12^2 = 13^2$.



Geometrically, for any right triangle, the sum of the areas of the squares on the legs is equal to the area of the square on the hypotenuse.

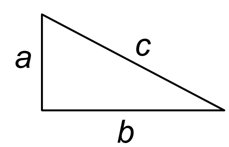


$A_1 + A_2 = A_3$

Slogan Form of the Pythagorean Theorem

Important theorems often have slogan forms. Slogan forms are useful in that they are easy to remember and they trigger recalling the full statement of the theorem. However, the slogan form of a theorem should not be confused with a correct statement of the theorem.

The slogan form of the Pythagorean Theorem is:



"*a* squared plus *b* squared equals *c* squared"

or

$a^2 + b^2 = c^2$.

While easily remembered, this statement is at best an incomplete statement of the Pythagorean Theorem. There is no reference to a right triangle or identification of variables. Here are two more complete statements of the Pythagorean Theorem:

1. For a right triangle with legs of length a and b and hypotenuse of length c, a squared plus b squared equals c squared.

2. For a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

Converse of the Pythagorean Theorem

The converse of a statement is created by interchanging the hypothesis and the conclusion. In other words, the converse of the statement "if A then B" is the statement "if B then A."

Statement: If n is an even integer, then $n + 1$ is an odd integer. (TRUE)
Converse: If $n + 1$ is an odd integer, then n is an even integer. (TRUE)

Statement: If n is a divisible by 9, then n is divisible by 3. (TRUE)
Converse: If n is a divisible by 3, then n is divisible by 9. (FALSE)

From the examples above, we see that the converse of a true statement may or may not be true.

The converse of the Pythagorean Theorem is **true**. If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle.

Example: If the lengths of the sides of a triangle are 3, 4, and 5 units respectively, then the triangle is a right triangle, because $3^2 + 4^2 = 5^2$.

Pythagorean Triples

A Pythagorean triple consists of three positive integers a, b, and c, such that $a^2 + b^2 = c^2$. We sometimes write the triple as (a, b, c).

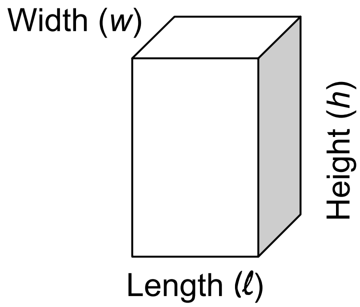
Some common triples are:

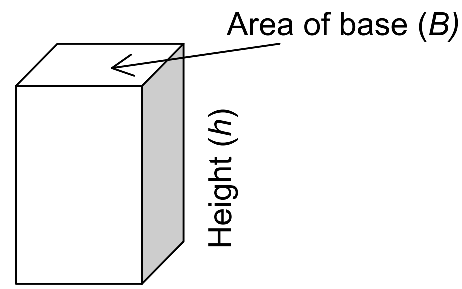
(3, 4, 5)	(5, 12, 13)	(8, 15, 17)	(7, 24, 25)
because	because	because	because
$3^2 + 4^2 = 5^2$ $9 + 16 = 25$ $25 = 25$	$5^2 + 12^2 = 13^2$ $25 + 144 = 169$ $169 = 169$	$8^2 + 15^2 = 17^2$ $64 + 225 = 289$ $280 = 289$	$7^2 + 24^2 = 25^2$ $49 + 576 = 625$ $625 = 625$

Recognizing a Pythagorean triple in a problem makes calculations easier. For example, if you know the sides of a triangle are 3, 4, and 5, then you know it is a right triangle. Furthermore any triple of the form $(3k, 4k, 5k)$, where k is a positive integer, is also a Pythagorean triple. Thus for instance (6, 8, 10) is a Pythagorean triple, so that a triangle with sides 6, 8, and 10 is a right triangle.

VOLUME

Volume of a Rectangular Prism





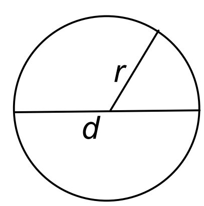
Surface Area	Volume
Find the area of each rectangular face. $SA = \ell w + \ell w + wh + wh + \ell h + \ell h$ $SA = 2\ell w + 2wh + 2\ell h$ $SA = 2(\ell w + wh + \ell h)$	Multiply the area of the base (B) by the height. $V = \ell wh$ or $V = Bh$

Formulas for Circles

Let r = radius of a circle.
Let d = diameter of a circle.

Circumference: $C = \pi d$ or $C = 2\pi r$

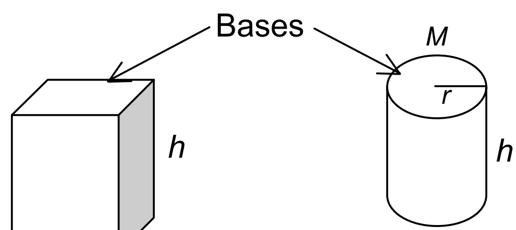
Area: $A = \pi r^2$



Volume of a Cylinder

Recall that the volume of a prism is the product of the area of the base and its height. We apply this relationship to find the volume of a cylinder, which has a circular base.

Let r = radius of the circular base.
Let h = height.



$$V_{cylinder} = Bh$$

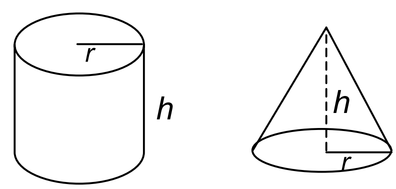
$$\text{Area of base } (B) = \pi r^2$$

Therefore, $V_{cylinder} = \pi r^2 h$

Volume of a Cone

Through experimentation, we observe that the volume of a cone is $\frac{1}{3}$ of the volume of a cylinder with the same height and base.

Let r = radius of the circular base
Let h = height



$$V_{cone} = \frac{1}{3}Bh$$

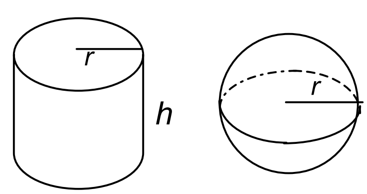
Area of base (B) = πr^2

$$V_{cone} = \frac{1}{3}\pi r^2 h$$

Volume of a Sphere

Through experimentation, we observe that the volume of a sphere is $\frac{2}{3}$ of the volume of a cylinder whose diameter and height are the same as the diameter of the sphere.

Let r = radius of the sphere and cylinder,
$2r$ = height (h) of cylinder.



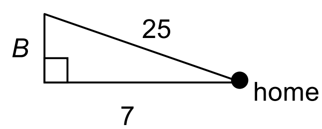
$$V_{cylinder} = \pi r^2 (2r) = 2\pi r^3$$

$$V_{sphere} = \frac{2}{3} \bullet 2\pi r^3 = \frac{4}{3}\pi r^3$$

Using Algebra to Solve Geometry Problems
When solving geometry problems algebraically: • Read the words in the problem one phrase at a time. • Record what you know and what you need to find in a picture. • Use symbols and numbers to write an equation. • Solve the equation. • Answer the question in words.

Example 1: To get from home to work every day, Sam drives 7 miles west on Avenue A, and then drives north on Avenue B. He knows that the straight-line distance from his home to his place of work is about 25 miles. How many miles is his drive north on Avenue B?

Picture (not to scale)



Symbols

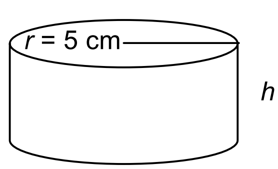
$$A^2 + B^2 = C^2$$
$$7^2 + B^2 = 25^2$$
$$49 + B^2 = 625$$
$$B^2 = 625 - 49$$
$$B^2 = 576$$
$$B = 24$$

Words

He drives 24 miles north on Avenue B.

Example 2: A cylinder has a radius of 5 cm. and a volume of 100π cu. cm. Find its height.

Picture



Symbols

$$V = \pi r^2 h$$
$$100\pi = \pi(5^2)h$$
$$100 = 25h$$
$$4 = h$$

Words:

The height is 4 cm.

TRANSFORMATIONS OF THE PLANE

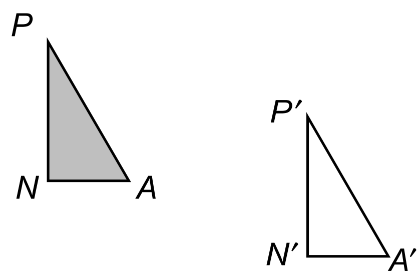
Transformations of the Plane

A transformation of the plane is a function that takes the plane to itself.

The input values are points in the plane. The output values (called the image of the transformation) are also points in the plane.

A transformation can be viewed as a mapping of input values to their corresponding images or output values.

In this figure, shaded triangle ΔPAN represents input values of a transformation and unshaded triangle $\Delta P'A'N'$ represents its image (output values).



The prime symbol (an apostrophe-like symbol) is often used to distinguish points in an original figure (input values) from their images (output values).

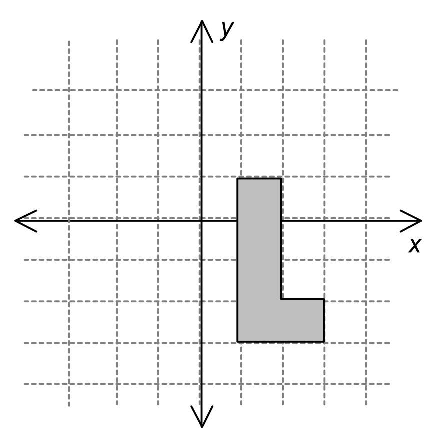
We use the arrow notation P → P' (read "point *P* is taken to point *P* prime" or "*P* maps to *P* prime") to indicate that the image of the point P under the transformation is P'.

In a coordinate plane, we use the coordinates to describe the transformation as in the following example.

The reflection in the *y*-axis maps the shaded L-figure to a backwards L-figure. For this transformation, the image of $P = (x, y)$ is $P' = (-x, y)$. We use the arrow notation

$$(x, y) \rightarrow (-x, y)$$

for this transformation.



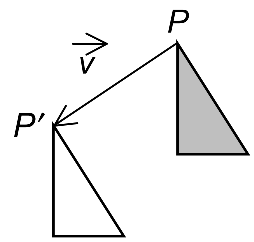
Translations, Rotations, and Reflections

Translations, rotations, and reflections are transformations of the plane that preserve distance between points.

A translation is a transformation that shifts all points the same distance and in the same direction.

This translation maps P to P' ($P \rightarrow P'$) (read "P maps to P prime").

The vector $\vec{v}$ shows the shift.

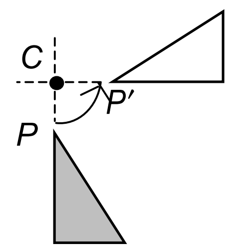


A rotation of a plane is a transformation that turns it through a given angle about a given point. The given point is called the center point of rotation. The given angle is called the angle of rotation.

This rotation maps P to P' ($P \rightarrow P'$).

Point C is the center point of the rotation.

The angle of rotation is 90° (or a quarter counterclockwise).

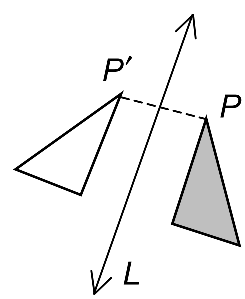


The reflection of a plane through a line L is the transformation that takes each point to its mirror image on the other side of L.

This reflection maps P to P' ($P \rightarrow P'$).

Line L is the line of reflection.

Line L is the perpendicular bisector of $\overline{PP'}$.

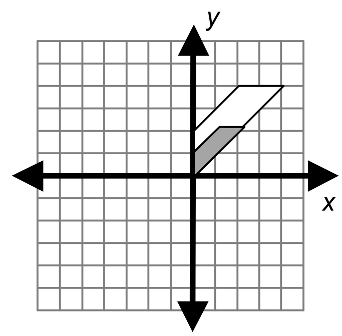


Translations, reflections, and rotations preserve distances between points. Further, translations, rotations, and reflections

- map lines to lines,
- map line segments to line segments of the same length,
- map parallel lines to parallel lines, and
- map angles to angles of the same measure.

Dilations of the Plane

A <u>dilation</u> is a transformation that moves each point along the ray through the point originating at a fixed center, multiplying distances from the center by a common scale factor.



The transformation of the plane to the right has a center at the origin and scale factor of 2. We represent the dilation as $(x, y) \rightarrow (2x, 2y)$.

Dilations share many but not all of the properties of translations, rotations, and reflections. Dilations

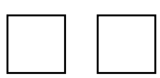
- map lines to lines (in fact to lines with the same slope),
- map parallel lines to parallel lines (that is, they preserve parallelism),
- map angles to angles of the same measure.

However, dilations DO NOT, in general, preserve distances. The only dilation with the center at the origin that preserves distances is the identity transformation $(x, y) \rightarrow (x, y)$, which has scale factor $s = 1$.

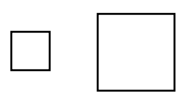
Congruence and Similarity

Two figures in the plane are <u>congruent</u> if one can be moved to exactly cover the other by a sequence of translations, rotations, and reflections.

Congruent squares have the same side length, and any two squares with the same side length are congruent.

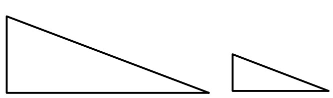


congruent

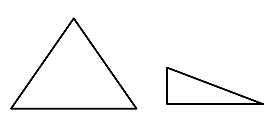


not congruent

Two figures in the plane are <u>similar</u> if one can be moved to exactly cover the other by a sequence of translations, rotations, reflections, and **dilations**. In similar figures, corresponding angles are congruent, and lengths of corresponding sides are proportional.



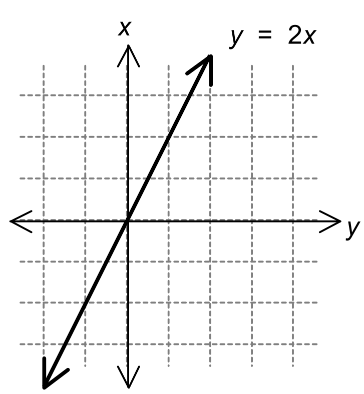
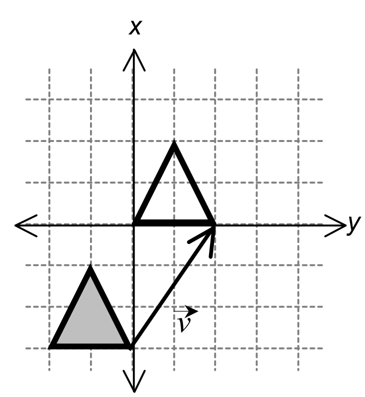
similar



not similar

Comparison of an Algebraic Function and a Geometric Function

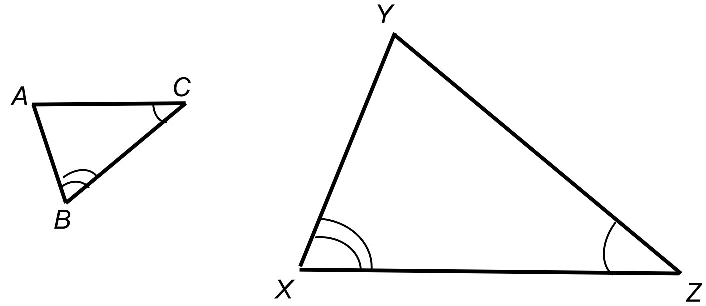
Functions arise in many different contexts. The way we think of them and even the language we use to talk about them may be quite different for different areas of math. Here we compare a typical <u>function</u> we might meet in an algebra course and a typical function (we call it a <u>transformation</u>) that we might study in geometry.

Area of mathematics	Algebra	Geometry
Name of function	Linear function	Translation
Domain of function	Real number line	Coordinate plane
Rule	Multiply by 2	Translate 2 units to the right and 3 units up
Description with symbols	x maps to $2x$ $x \rightarrow 2x$ $y = 2x$	(x, y) maps to $(x + 2, y + 3)$ $(x, y) \rightarrow (x + 2, y + 3)$ $(x', y') = (x + 2, y + 3)$
Graph		
Graph interpretation	The x-coordinates represent the inputs and the y-coordinates represent the outputs. The set of all input-output pairs is represented by the line.	A figure (shaded triangle - input) and its image (unshaded triangle - output) illustrate what happens to a typical figure in the plane. The translation vector indicates the direction and distance each point is moved.

Angle-Angle Similarity Criterion

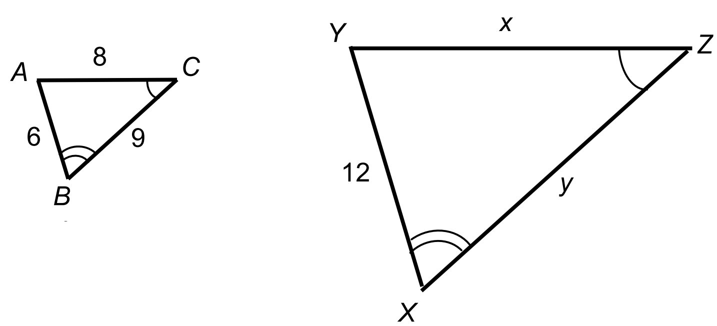
AA Criterion: If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

In this figure, the triangle *ABC* is similar to the triangle *YXZ*, that is, $\Delta ABC \sim \Delta YXZ$.



Finding Side Lengths of Similar Triangles

In this figure, $\Delta ABC \sim \Delta YXZ$. There are two basic ways to set up proportions to find missing side lengths in similar triangles.



Method 1: Establish values of ratios of corresponding segments **between** the two figures.

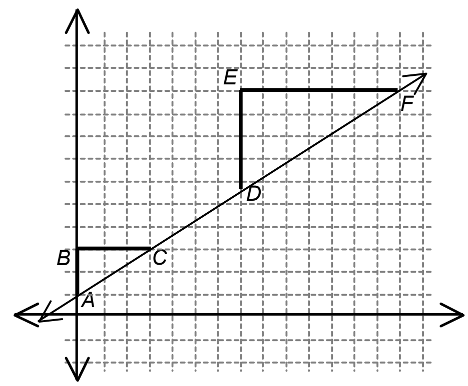
$$\frac{6}{12} = \frac{8}{x} \rightarrow x = 16 \quad \text{and} \quad \frac{6}{12} = \frac{9}{y} \rightarrow y = 18$$

Method 2: Establish values of ratios of corresponding segments **within** the two figures.

$$\frac{8}{6} = \frac{x}{12} \rightarrow x = 16 \quad \text{and} \quad \frac{9}{6} = \frac{y}{12} \rightarrow y = 18$$

Slope and Similarity

To find the slope of the line on the right, we might choose two points on the line, say A and C, and calculate the ratio of vertical change to horizontal change (the "rise over the run"). In this case we get:



$$\text{slope} = \frac{|AB|}{|BC|}$$

If we choose two other points, say D and F, and calculate the same ratio of rise over the run, we get:

$$\text{slope} = \frac{|DE|}{|EF|}$$

How do we know these two ratios are the same? One way to see this is to apply the AA Criterion to establish that the triangles ΔABC and ΔDEF are similar. Here is a two-column proof of similarity:

Statement	Reason
$\overleftrightarrow{BC} \parallel \overleftrightarrow{EF}$	Horizontal lines are parallel.
$\lvert\angle BCA\rvert = \lvert\angle EFD\rvert$	If two parallel lines are cut by a transversal, then corresponding angles have equal measure
$\lvert\angle ABC\rvert = \lvert\angle DEF\rvert$	All right angles have equal measure
$\Delta ABC \sim \Delta DEF$	Angle-Angle Similarity Criterion

Since ΔABC and ΔDEF are similar, their corresponding sides are proportional. Therefore:

$$\frac{|AB|}{|BC|} = \frac{|DE|}{|EF|}$$

We conclude that the slope ratios obtained above are the same, no matter which pair of points we use on the line to calculate the slope.

In other words, the slopes of any two line segments lying on the same non-vertical line are the same.

STATISTICS

Categorical Data Versus Numerical Data

Categorical data is data sorted into categories, such as colors, ranges of measurements, or other attributes of the data. Generally there are only finitely many categories.

Categorical survey questions are used to collect categorical data. Responses to these questions are usually in words.

Example of a categorical survey question: “What type(s) of pet(s) do you own?” Answers may include, dog, cat, bird, no pets, etc.

Numerical data is data consisting of numbers. Measurement data is numerical data that comes from making measurements.

Numerical survey questions are used to collect numerical data. Numerical data sometimes comes from counting. It can sometimes come from measurements.

Example of a numerical survey question: “How many dogs do you own?” Answers may include 0, 1, 2, etc.

Example of a measurement survey question: “How much does your pet weigh?” Answers may include 5 oz, 18 kg, 60 lbs, etc.

Categorical Variables and Frequencies

A categorical data set based on a population is sometimes referred to as a “categorical variable”. Technically, a categorical variable is a function that assigns to each member of the population a category. Since a categorical data set has only a finite number of categories, a categorical variable can assume only a finite number of values, namely, the finitely-many possible categories.

Example of a population: students in a class.

Example of a categorical variable based on this population: a function that assigns pet type to each student (assigning something like fish, hamster, bird, no pet, etc. to each student).

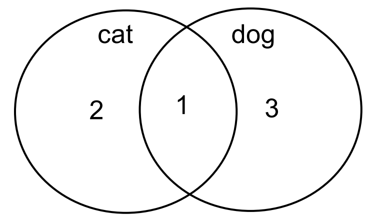
The frequency of a category is the number of times that that the variable assumes that category as a value, that is, the number of members of the population belonging to that category.

Example for frequencies based on the categories above: 3 students have fish, 4 have a hamster, 1 has a bird, 7 have no pets, etc.

Venn Diagrams

A Venn diagram is a pictorial way to represent relationships between sets, in which the sets are represented by regions in the plane.

Example: Amir asked 10 students in his class whether they owned a dog or a cat. Their responses are shown in the Venn diagram to the right.



- ***How many have a cat?*** The "cat" set has 2 + 1 = 3 students.
- ***How many have a dog?*** The "dog" set has 3 + 1 = 4 students.
- ***How many have a cat, but not a dog?*** The part of the "cat" set outside the "dog" set has 2 students.
- ***How many have a dog, but not a cat?*** The part of the "dog" set a outside the "cat" set has 3 students.
- ***How many have both a cat and a dog?*** The region inside both the "cat" set and "dog" set has 1 student.
- ***How many have neither a cat nor a dog?*** The number of students without a cat or a dog is 10 – 6 = 4 students.

Bivariate Data and Two-Way Frequency Tables

A frequency table is a table that lists items and the number of times they occur in a data set.

Bivariate data is data that has two variables (based on the same population).

A two-way table is a table that displays bivariate categorical data, in which the rows correspond to the categories of one variable, and the columns correspond to the categories of the other.

Example: Emilio asked 10 students in his class whether they owned a dog or a cat. Here is a table of the data.

	Dog	No Dog	TOTAL
Cat	1	2	3
No Cat	3	4	7
TOTAL	4	6	10

NOTE: When constructing a two-way frequency table, include an extra column and an extra row for totals. These totals appear as the denominators of the fractions representing the various percents that answer the following typical questions.

- ***What percent of the students own a dog?*** $\frac{4}{10} = 40\%$
- ***What percent of the students do not own a dog?*** $\frac{6}{10} = 60\%$
- ***What percent of the students own a cat?*** $\frac{3}{10} = 30\%$
- ***What percent of the students do not own a cat?*** $\frac{7}{10} = 70\%$
- ***What percent of the students own both a cat and a dog?*** $\frac{1}{10} = 10\%$
- ***What percent of the students own neither a cat nor a dog?*** $\frac{4}{10} = 40\%$
- ***What percent of the students who do not own a cat, own a dog?*** $\frac{3}{7} \approx 43\%$
- ***What percent of the students who own a dog, do not own a cat?*** $\frac{3}{4} = 75\%$
- ***What percent of the students who do not own a dog, own a cat?*** $\frac{2}{6} \approx 33\%$
- ***What percent of the students who own a cat, do not own a dog?*** $\frac{2}{3} \approx 66\%$
- ***What percent of the students who do not own a cat, do not own a dog?*** $\frac{4}{7} \approx 57\%$
- ***What percent of the students who do not own a dog, do not own a cat?*** $\frac{4}{6} \approx 66\%$

Relative Frequency Tables

Example: Emilio asked 10 students in his class whether they owned a dog or a cat. Here is a table of the data.

	10 Students (A through J)									
	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
Dog	Yes	No	No	Yes	Yes	No	Yes	No	No	No
Cat	Yes	No	No	No	No	No	No	Yes	No	Yes

A relative frequency table is a variant of a frequency table that lists items and the proportion (or percent) of times they occur.

Using the data above, we may construct a relative frequency table in different ways.

Table 1: Cats and Dogs
(totals based on dog / no dog)

	Dog (n = 4)	**No Dog** (n = 6)
Cat	$\frac{1}{4} = 25\%$	$\frac{2}{6} \approx 33\%$
No Cat	$\frac{3}{4} = 75\%$	$\frac{4}{6} \approx 67\%$
TOTAL	100%	100%

Table 2: Cats and Dogs
(totals based on cat / no cat)

	Dog	**No Dog**	**TOTAL**
Cat (n = 3)	$\frac{1}{3} \approx 33\%$	$\frac{2}{3} \approx 67\%$	100%
No Cat (n = 7)	$\frac{3}{7} \approx 43\%$	$\frac{4}{7} \approx 57\%$	100%

Creating Relative Frequency Tables

Creating accurate relative frequency tables requires the ability to think concisely about various subgroups in a sample. Each row or column in a relative frequency table measures a different subgroup using one of the "total" values found in the frequency table. In the relative frequency tables in lesson 10.1 and elsewhere, we include "n = ___" to help identify which value to use as a denominator in each row or column of the relative frequency table.

The following model can be used as a graphic organizer to help create accurate relative frequency tables. Each letter in the frequency table below represents a frequency count.

Frequency Table

	Rock	**Rap**	**Total**
7th grade	a	b	c
8th grade	d	e	f
Total	g	h	*

What is the sum in this box? This represents the sum of all responses, so the sum is equal to $a + b + d + e$, $c + f$, or $g + h$. All three of these expressions should shave a sum equal to 100% of the responses.

Relative Frequency Table #1

	Rock	**Rap**	**Total**
7th grade ***n* = *c***	$\frac{a}{c}$	$\frac{b}{c}$	100%
8th grade ***n* = *f***	$\frac{d}{f}$	$\frac{e}{f}$	100%

Relative Frequency Table #2

	Rock ***n* = *g***	**Rap** ***n* = *h***
7th grade	$\frac{a}{g}$	$\frac{b}{h}$
8th grade	$\frac{d}{g}$	$\frac{e}{h}$
Total	100%	100%

Association and Causality
One of the goals of statistics is to determine whether two variables are related and, if so, to determine the strength of the relationship. If there is evidence of a relationship, we say the variables are associated, or there is an association between the variables. In the case of two numerical variables, we can look for an association by creating a scatter plot, that is, by graphing data pairs in a coordinate plane. If the data points cluster on a curve, there is likely an association between the variables, and the equation of the curve suggests the functional relationship between the variables. If the data points cluster along a straight line, we say the data suggests a linear association between the variables. The fact that there is an association between two variables does not mean that a change in one variable might cause a change in the other. Other factors may cause changes in both variables simultaneously. The fact that there is a positive association between smoking and lung cancer does not *by itself* imply that smoking causes lung cancer. Further statistical and medical evidence is required to make that case. By the same token, the fact that there is a positive association between atmospheric carbon and global warming does not per se imply that increased carbon in the atmosphere causes global warming. Further statistical and physical evidence is required to establish causality.

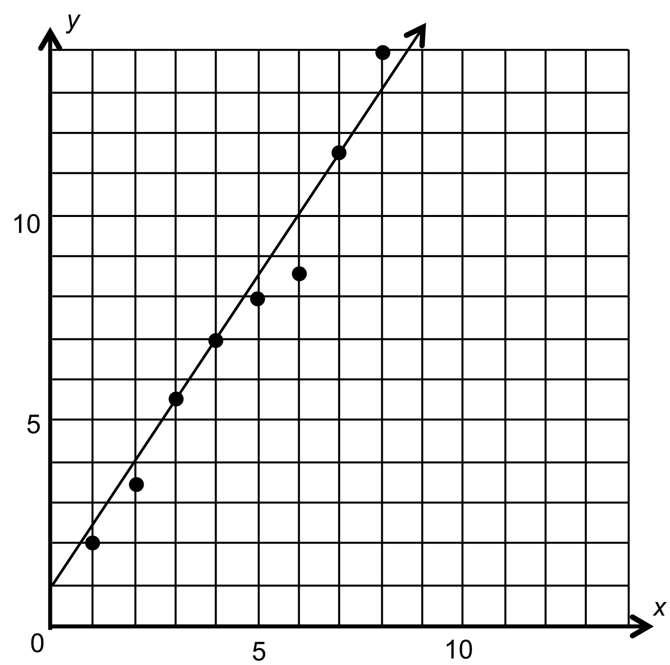
Lines of Best Fit

A line of best fit for a scatter plot is a straight line that best represents (in some sense) the data points in the scatter plot.

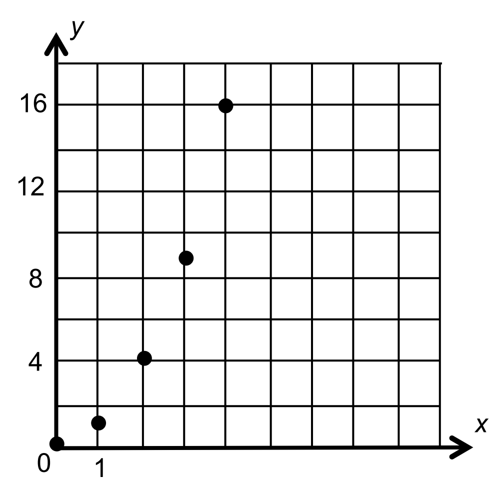
Example: When the data in the table below is graphed in a scatter plot, the data points cluster along a straight line. We conclude that there is likely a linear association between x and y.

One possible such line may be estimated by the equation graphed below, $y = \frac{3}{2}x + 1$. Using a graphing calculator, another estimated equation is given as $y = 1.6x + 0.3$ (not graphed).

x	*y*
1	2
2	3.5
3	5.5
4	7
5	8
6	8.5
7	11.5
8	14



Not all associations are linear. Here is an example of a scatter plot of bivariate data that appears to have a nonlinear association.



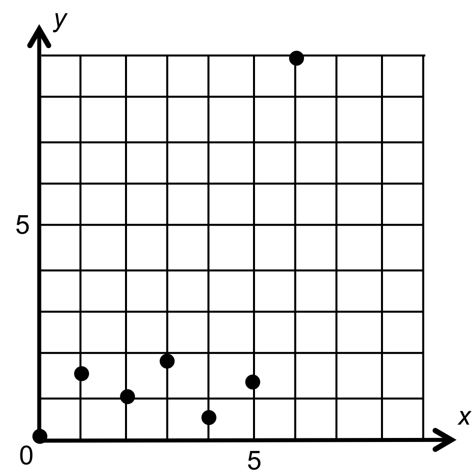
Outliers

An <u>outlier</u> of a data set is a data value that is unusually small or unusually large relative to the overall pattern of values in the data set.

Example: In a 6th grade classroom, students were asked how many pets they had. All students but one replied with numbers of pets that ranged from 0 to 8. That one pet owner said she had 40 fish. This number of fish appears to be an outlier, because it is unusually large compared to the other numbers of pets.

In the scatter plot to the right, the data point (6, 9) is a potential outlier. Its y-coordinate 9 appears to be unusually large compared to the other y-coordinates.

Outliers can create the illusion that an association exists when one does not. They can also distract us from seeing an association when there clearly is one. The examples in the Lines of Best Fit lesson Lesson 10.3, page 22 illustrate these phenomena.



This page was left intentionally blank.

9 7 8 1 6 1 4 4 5 2 2 5 6